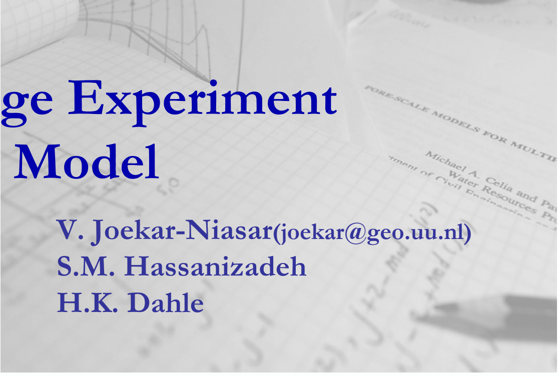
Dynamics of Two-Phase Drainage Experiment Using a Pore-Network Model



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Objectives

Development of a new dynamic pore-network model for simulating drainage for angular cross sections. This network is applied for studying transient capillary pressure-saturation curves. Traditional two-phase flow models assume: $< P^{nw} > - < P^{w} >= P^c(S_w)$

However, under dynamic conditions, the difference in phase pressures is known to be a function of time rate of saturation change. (Hassanizadeh & Gray,1990)

$$P^{nw} - P^{w} = P^c_{static} - \tau \frac{dS_w}{dt}$$

Assumptions and Issues

- Pore-network is 3-D Regular and structured (coordination number of 6).
- Pore throats have square cross-sections.
- Pore bodies are assumed as cubes.
- Volume of pore throats is negligible compared to pore bodies volume.
- Resistance to flow is assigned to pore throats.
- Linear relationship between flux and pressure gradient at pore-scale is assumed.
- There is a local capillary pressure within each pore body that is a function of local saturation.
- The phase pressures are solved for each phase separately.
- Wetting phase is assumed always connected to its reservoir.
- Dirichlet boundary conditions applied on both ends.

Governing Equation and Local Rules

Defining a saturation weighted pressure at each pore body as $\bar{P}_i = S^w_i P^w_i + S^n_i P^n_i$

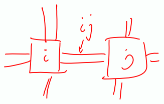
Thus: $P^n_i = \bar{P}_i + S^w_i P^c_i \qquad P^w_i = \bar{P}_i - S^n_i P^c_i$

Governing equation will in each pore body will be

$$\sum_{j\in N_i} [(K^w_{ij} + K^n_{ij})(\bar{P}_i - \bar{P}_j) + (K^n_{ij} S^w_i - K^w_{ij}(1 - S^w_i))P^c_i + (K^w_{ij}(1 - S^w_j) - K^n_{ij} S^w_j)P^c_j] = 0$$

Conductivity rules

Case 1- Only wetting phase is present in the pore throat.



$$K^n_{ij} = 0$$

$$K^w_{ij} = \frac{\pi}{8\mu^w l_{ij}}(r^{eff}_{ij})^4 \qquad r^{eff}_{ij} = \sqrt{\tfrac{4}{\pi}} r_{ij}$$

Case 2- Both phases are present in the pore throat.

$$K^n_{ij} = \frac{\pi}{8\mu^n l_{ij}}(r^{eff}_{ij})^4 \qquad r^{eff}_{ij} = \tfrac{1}{2}\left(\sqrt{\tfrac{r^2_{ij} - (4-\pi) r^{c2}_{ij}}{\pi}} + r_{ij}\right)$$

$$K^w_{ij} = \frac{4-\pi}{\beta\mu^w l_{ij}}(r^c_{ij})^4 \qquad r^c_{ij} = \frac{\sigma}{P^c_{ij}}$$

Invasion into a pore throat can happen when the local capillary pressure at a pore body is larger than the entry capillary pressure of the connected pore throat.

$$P^c_i > P_{c_{entry}} = \tfrac{\sigma}{r}\left(1 + \tfrac{\sqrt{\pi}}{2}\right)$$

Saturation update

- **Explicit saturation update:** $V_i \frac{\Delta S^w_i}{\Delta t} - \Sigma Q^n_{ij} = 0$

- **Implicit saturation update:**

$$V_i \tfrac{(S^w_i)^{n+1} - (S^w_i)^n}{\Delta t} +$$
$$\sum_{j\in N_i} \left[\left(\tfrac{K^w_{ij}}{K^{total}_{ij}} Q^{total}_{ij}\right)^n - \left(\tfrac{K^w_{ij} K^n_{ij}}{K^{total}_{ij}} P^{c'}_{ij,s}\right)^n \left((S^w_i)^{n+1} - (S^w_j)^{n+1}\right) \right] = 0$$

Conclusion

More detailed features are included in the dynamic pore network model; phase pressures are solved separately and capillary pressure in each pore is defined as a function of local saturation.

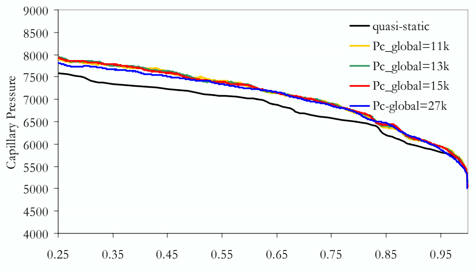
The model can be used for different viscosity ratios and different capillary numbers.

Increasing pressure difference at boundaries will increase flooding efficiency.

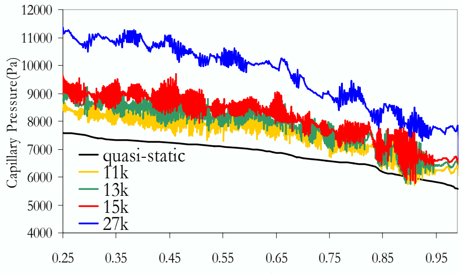
Increasing pressure difference at boundaries will change interface pattern from fingering to a smooth pattern.

Damping coefficient is τ obtained as a function of saturation.

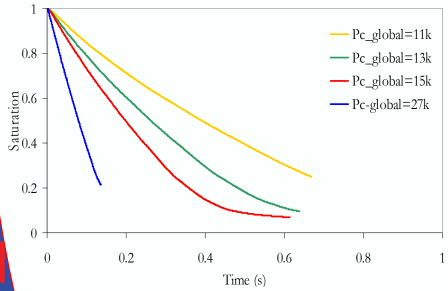
Results



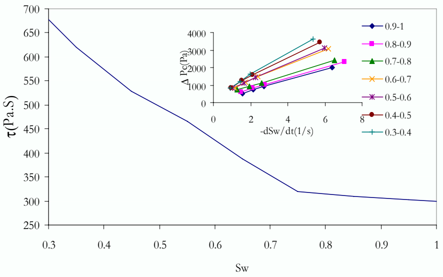
Average of saturation-weighted Pc



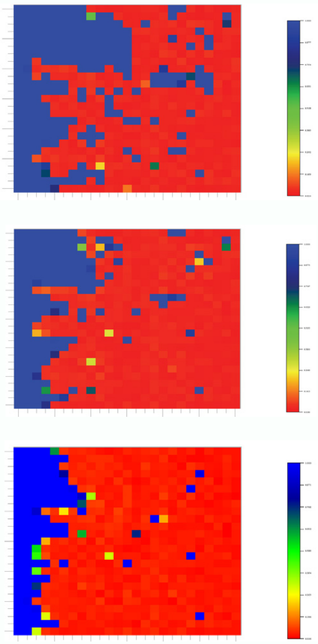
Saturation-weighted average of phase pressures $< P_{nw} > - < P_w >$



Saturation change with time



Variation of τ with saturation



Boundary Pc=12kPa

Boundary Pc=30kPa

Boundary Pc=100kPa

Effect of boundary pressure difference on the front pattern and flooding efficiency