The use of spatio-temporal correlation to forecast critical transitions

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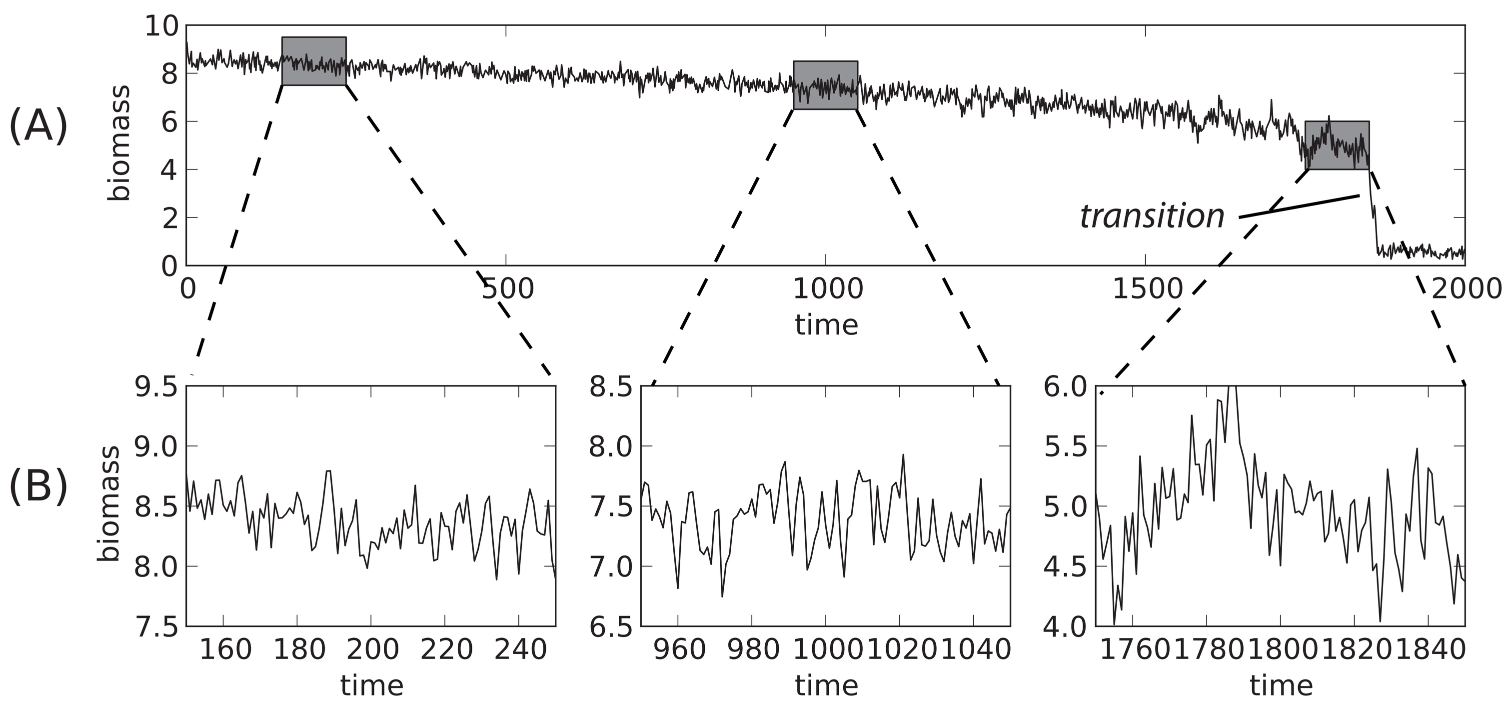
Introduction

Complex dynamical systems may have critical thresholds at which the system shifts abruptly from one state to another (Scheffer et al., 2009). Forecasting the timing of critical transitions is of paramount importance, because critical transitions are associated with a large shift in dynamical regime of the system. However, it is hard to forecast critical transitions, because the state of the system shows relatively little change before the threshold is reached. Here we show how spatio-temporal autocorrelation can be used to significantly reduce the uncertainty in forecasts of critical transitions.

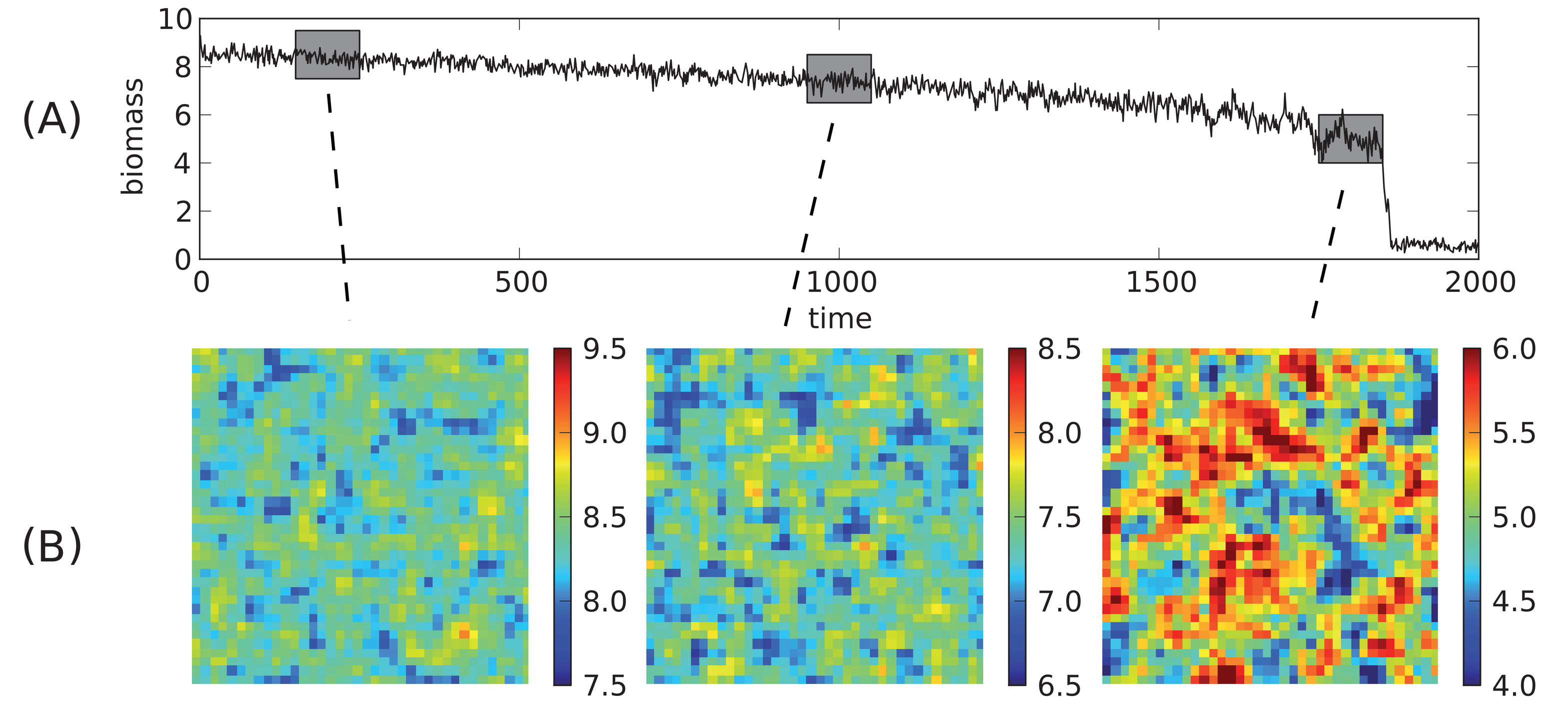
The system studied: spatially distributed logistic growth

$$\frac{dX_{i,j}}{dt} = rX_{i,j}\left(1-\frac{X_{i,j}}{k}\right) - c\,\frac{X_{i,j}^{2}}{X_{i,j}^{2}+1} + d(X_{i+1,j} + X_{i-1,j} + X_{i,j+1} + X_{i,j-1} - 4X_{i,j})$$

X_{ij} biomass at grid cell i, j, uncorrelated white noise added
r growth rate
k carrying capacity
c grazing rate, linearly increased over time
d dispersion rate



Artificial data set created with the model, linear increase of grazing rate over time causes very little decrease in biomass, until the critical transition is reached (A). Variance of biomass and correlation length (scale of variation) increases well before the transition is reached (B).



Patch size on maps (B) of biomass increases gradually before reaching the transition. The maps show also an increase in variance.

Sampling the spatio-temporal patterns

We sampled the artificial real-world (created above) using a regular sampling scheme, adding white noise and bias to mimic sampling error. This was done at a 50-timestep interval. From these samples, semivariance values at multiple separation distances were calculated representing the spatio-temporal patterns in biomass:

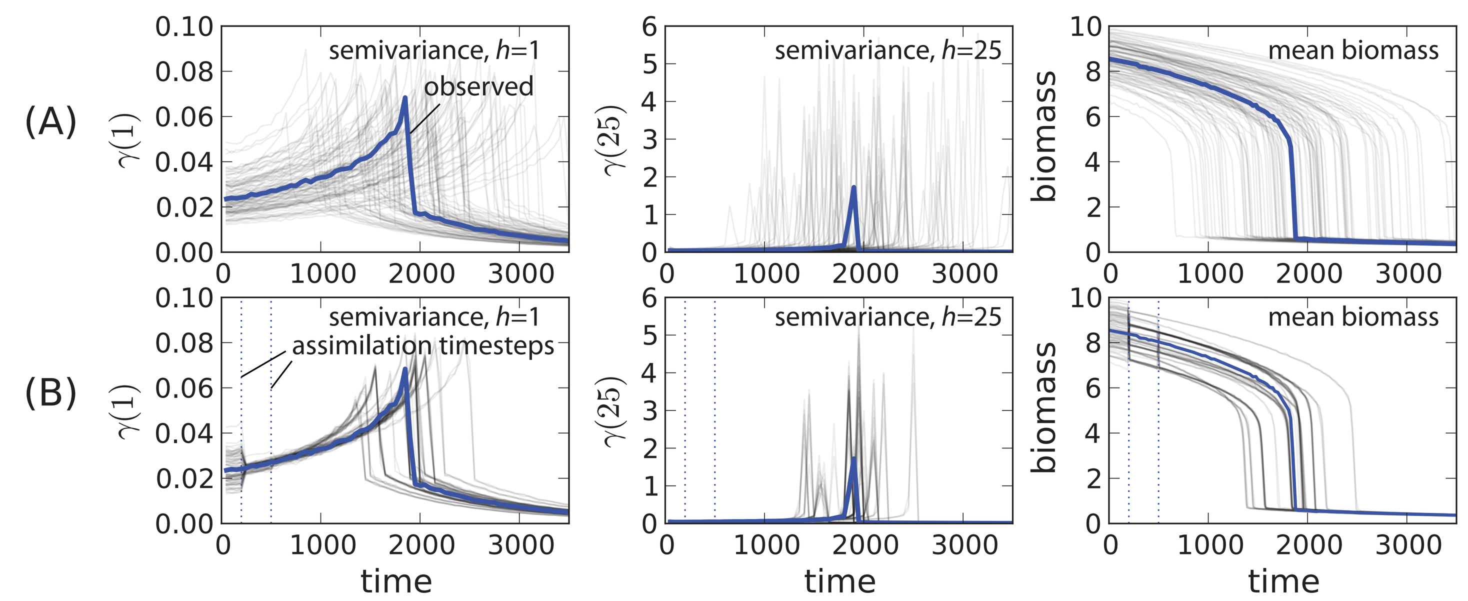
$$\gamma(h) = \frac{1}{N(h)} \sum (X(\mathbf{s}) - X(\mathbf{s}+h))^2$$

$\gamma(h)$ semivariance at separation distance h
$N(h)$ number of sample pairs with separation distance h
$X(\mathbf{s})$ biomass, **s** is spatial index

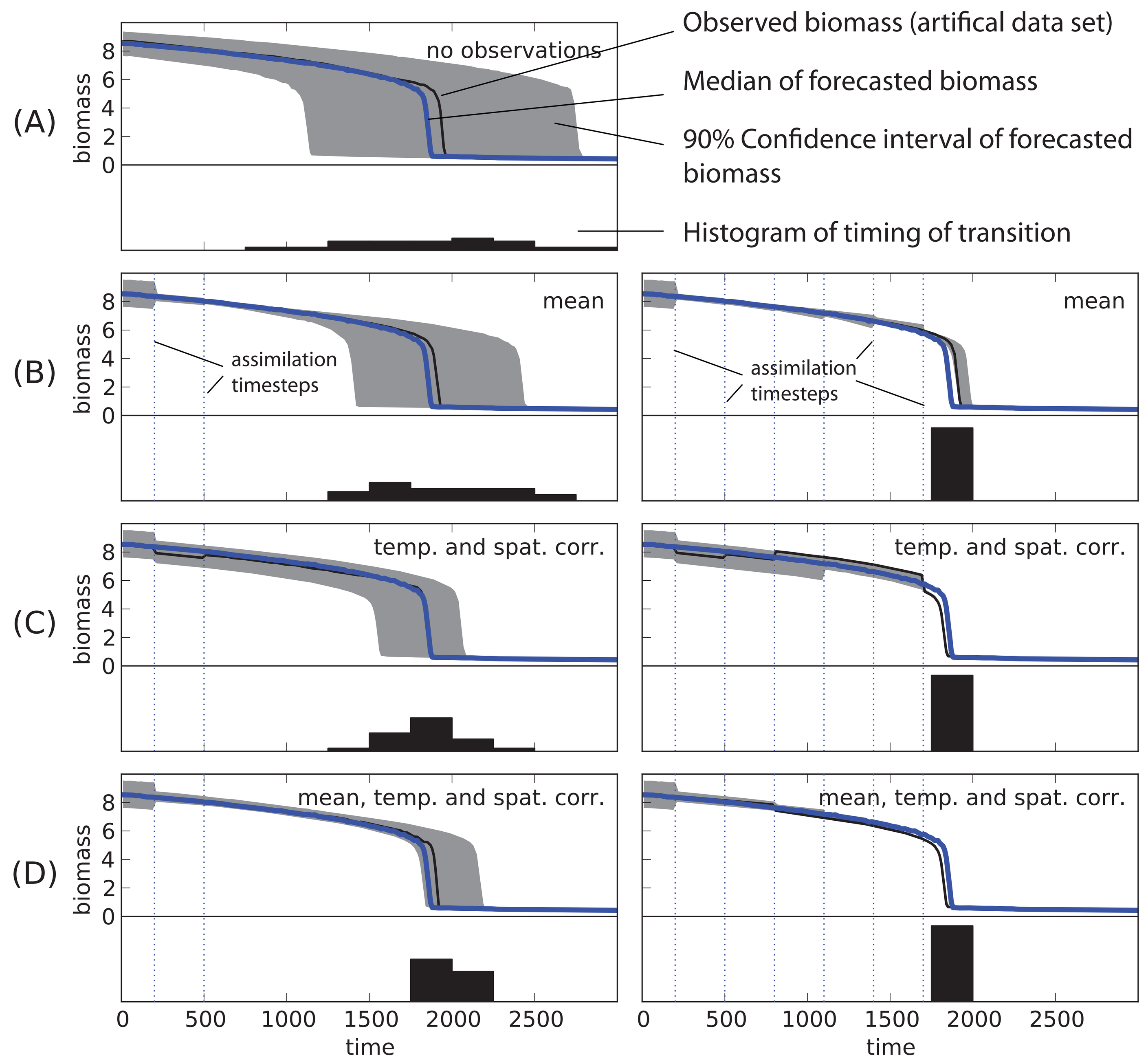
Forecasting the timing of the transition

The timing of the critical transition was forecasted by assimilating sampled semivariance data into the growth model. This was done with the Particle Filter (e.g., van Leeuwen, 2003). Prior distributions of all parameters and inputs were taken as uniform. The covariance matrix of the sampling error (of the semivariance values) required in the assimilation scheme was calculated using Monte Carlo simulation, for each assimilation time step.

Results & conclusions



Realizations (particles) of the growth model. Assimilating sampled semivariance values (B panels) reduces uncertainty in forecasted timing of the transition, compared to no assimilation (A panels).



The effect of the type of observational information used in the filter: (A), no data assimilation; (B) data assimilation using sampled mean biomass ('classical method'), (C) idem, using temporal and spatial semivariance, (D) idem, using all information. The use of spatia-temporal patterns results in signficantly lower uncertainty compared to the classical method (panel B). Thus, spatio-temporal patterns can better be used to predict transitions.

References
Scheffer et al., 2009. Early warning signals for critical transitions. Nature 461 (7260): 53-59.
van Leeuwen, 2003. A variance minimizing filter for large scale applications. Monthly Weather Review 131 (9): 2071-2084.