Towards quantifying uncertainties in travel-time tomography using the null space shuttle

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Introduction

A large part of the uncertainty in global travel time tomography is related to the non-uniqueness of the solution. We suggest to explicitly address the non-uniqueness and take the null space of the forward operator into account when analyzing model uncertainties. Deal and Nolet (1996) designed the null space shuttle to exploit components of the model null space, in combination with physical *a priori* information, to enhance the corresponding tomographic image (Deal et al., 1999). We generalize this technique and use the null space shuttle to investigate the uncertainties in classical travel time tomographic models. As an example we use the P-wave model MIT-P08 (Li et al., 2008), since this model is based on a more comprehensive data set than most other P wave models.

Data & their uncertainties

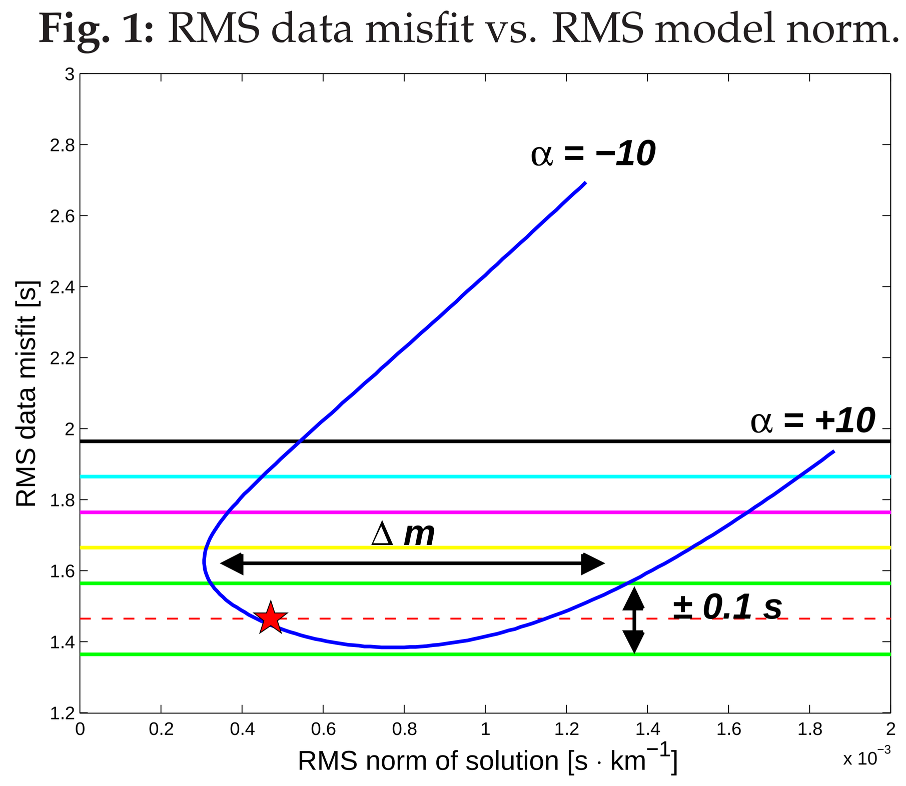
We use the same data set as Li et al. (2008), comprising millions of travel time residuals with respect to travel times computed from *ak135*. Available estimates of the random and systematic errors range from 0.3 s to 1.0 s for the ISC data. We use composite ray arguments to obtain a conservative estimate for the data uncertainty of 0.1 s, which we use as a tolerance on the data misfit.

- RMS of data vector $\mathbf{d} = 1.92\ s$
- RMS of data misfit $\mathbf{G}\mathbf{m}^{\mathbf{orig}} - \mathbf{d} = 1.46\ s$

Results

We follow the inversion procedure by (Li et al., 2008). All solutions were obtained after 100 iterations of the LSQR algorithm. Fig. 1 shows the RMS data misfit versus RMS norm of $\tilde{m}^{new}$ (slowness parameters only) for new solutions by setting $m_t = \tilde{m}^{orig}$ in Eq. 4. By changing α, we are able to minimize the norm of the solution or improve the data misfit. A large range of solutions exists that fits the data within a realistic average data uncertainty.

Fig. 1: RMS data misfit vs. RMS model norm.



Conclusions

The large range in model parameters (Fig. 1, 3) indicates that constraints from the travel time data on amplitude are lacking and prohibit robust inferences on thermochemical variations in the Earth from such models alone. While the lack of constraint on the amplitudes is qualitatively mentioned in many studies, the null space shuttle can put quantitative bounds on the amplitude range. However, any detailed interpretation of travel time tomography involving amplitudes needs to address the data uncertainty in much greater detail.

We find that independent of the regularization the short-wavelength structures in the model are mostly not constrained by the data. We conclude that 'high-resolution' travel time tomography provides useful insight into the structure of the Earth's mantle, but does not necessarily provide robust images on the length scale of the individual structures. Therefore, physical interpretations of velocity perturbations in the tomographic image should always be accompanied by additional information from other sources, such as studies of plate reconstructions, Benioff zones, etc.

Null space shuttle

Consider a test model m_t that may not explain the data. This model can be seen as the sum of the components lying in the range and in the null space of the forward operator, so that

$$m_t = m_t^{range} + m_t^{null} \tag{1}$$

m_t^{null} has no effect on the data misfit and can be found using the null space shuttle. Defining $d_t = Gm_t$ with d_t the synthetic data vector corresponding to m_t and G the forward operator, gives

$$d_t = Gm_t^{range} \tag{2}$$

as $Gm_t^{null} = \bar{0}$ by definition. Solving the inverse problem for m_t in Eq. 2 yields m_t^{range} and it is trivial to obtain m_t^{null} via Eq. 1. Due to the necessary regularization the solution to our inverse problem, and most others, is a compromise between the minimum norm and the least-squares solution. Therefore, we only get an estimate:

$$\tilde{m}_t^{range} = Ld_t = Rm_t \tag{3}$$

with L the inverse operator corresponding to the LSQR algorithm, R the resolution operator and $\tilde{m}_t^{range} \neq m_t^{range}$, although they are close. We define a new solution

$$\tilde{m}_{new} = \tilde{m}^{orig} + \alpha\tilde{m}_t^{null} \tag{4}$$

with $\tilde{m}^{orig}$ the original solution and α a scaling factor. Since $G\tilde{m}_t^{null}$ is not exactly zero, the new solution in Eq. 4 corresponds to a slightly different data misfit than the original solution. However, effects on the data misfit are small compared to presumed data uncertainties.

The null space shuttle is straightforward to implement:

Test model → Calculate synthetic data → Invert synthetic data → Calculate null space component → Add to original model → New model, comparable data misfit

Results — Range of model parameters

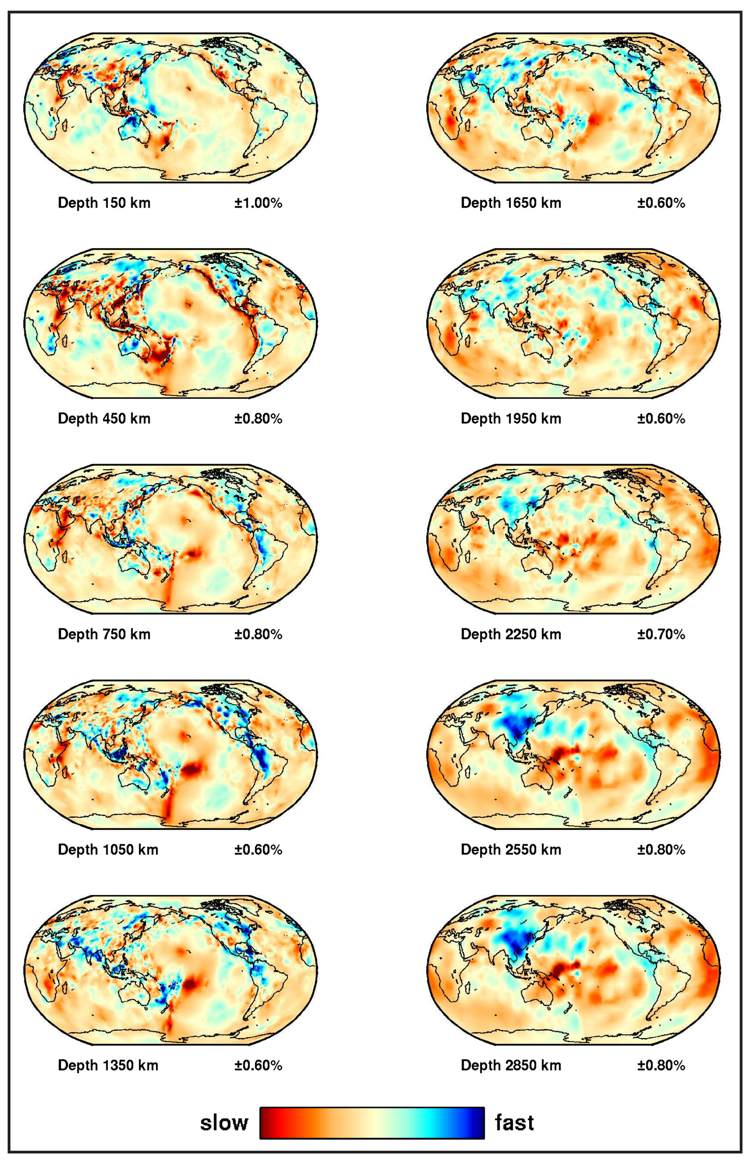


Fig. 2: Original solution MIT-P08.

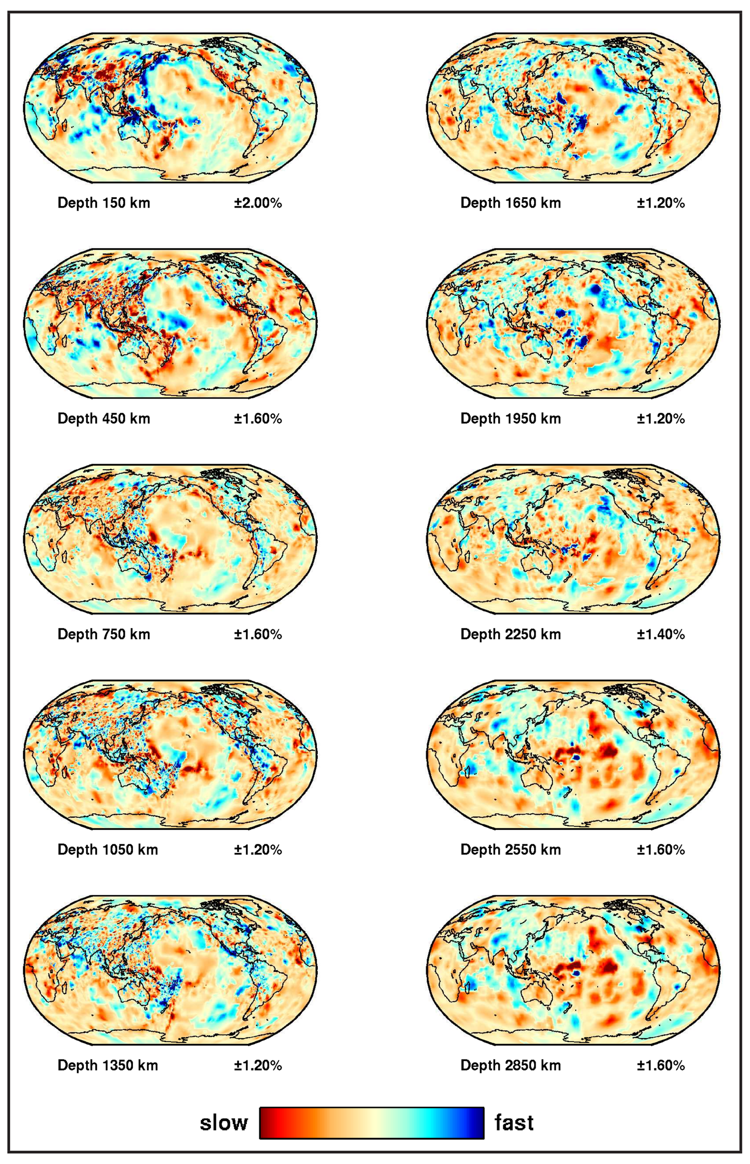


Fig. 3: Range of model parameters (Δm in Fig. 1). Note the different colour scale.

Results — Minimum-norm model

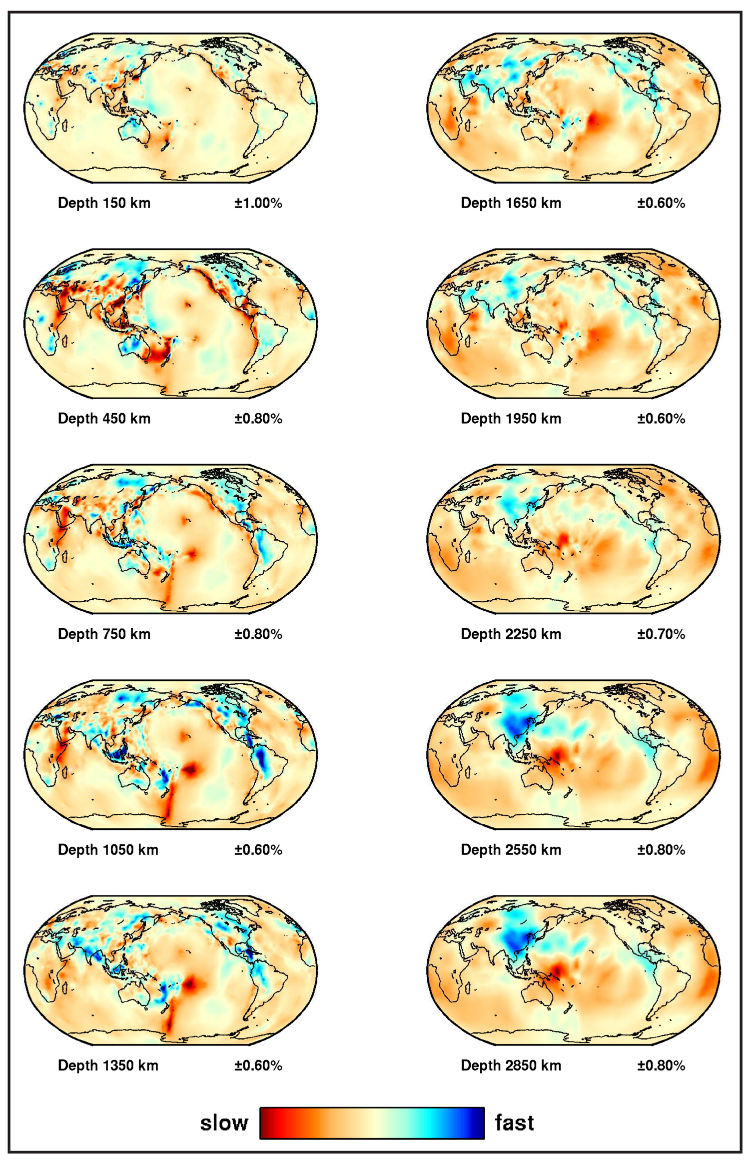


Fig. 4: Minimum-norm model.

For the chosen tolerance of the data misfit of 0.1 s, the model in Fig. 4 represents the model of velocity perturbations with the smallest norm required by the data (for the chosen regularization).

The differences between the models are mainly for relatively short wavelengths. The data do not constrain all of the model parameters at the high resolution of the model parameterization and therefore a large part of the short wavelength structure is introduced by the regularization, i.e. resides in the model null space.

References

Deal, M. M. and G. Nolet (1996). Nullspace shuttles. *Geophys. J. Int. 124*, 372–380.

Deal, M. M., G. Nolet, and R. D. van der Hilst (1999). Slab temperature and thickness from seismic tomography, 1. Method and application to Tonga. *J. Geophys. Res. 124*, 372–380.

Li, C., R. D. van der Hilst, E. R. Engdahl, and S. Burdick (2008). A new global model for P wave speed variations in Earth's mantle. *Geochem. Geophys. Geosyst 9*(5).

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