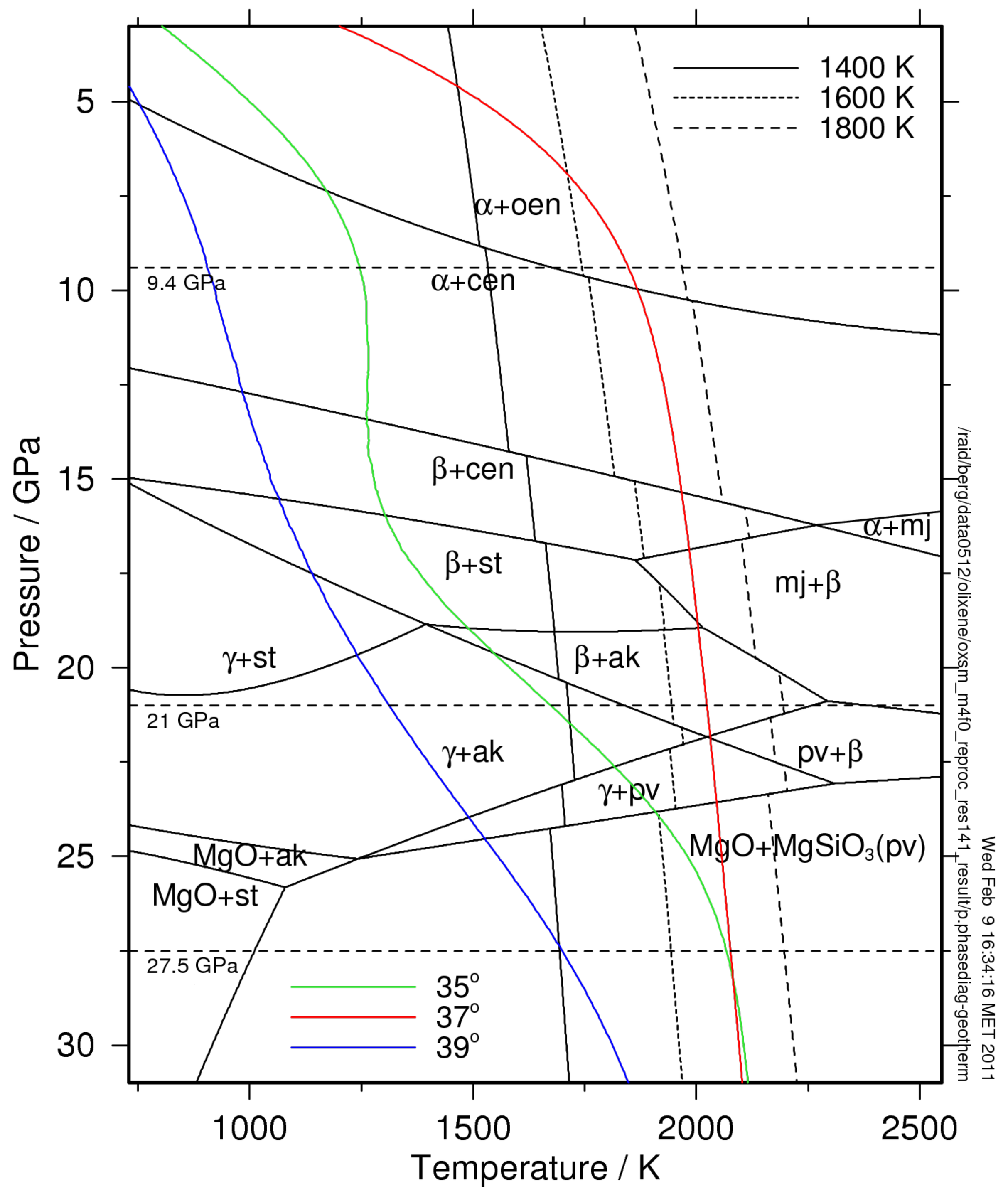
3D high resolution mineral phase distribution and seismic velocity structure of the transition zone: predicted by a full spherical-shell compressible mantle convection model

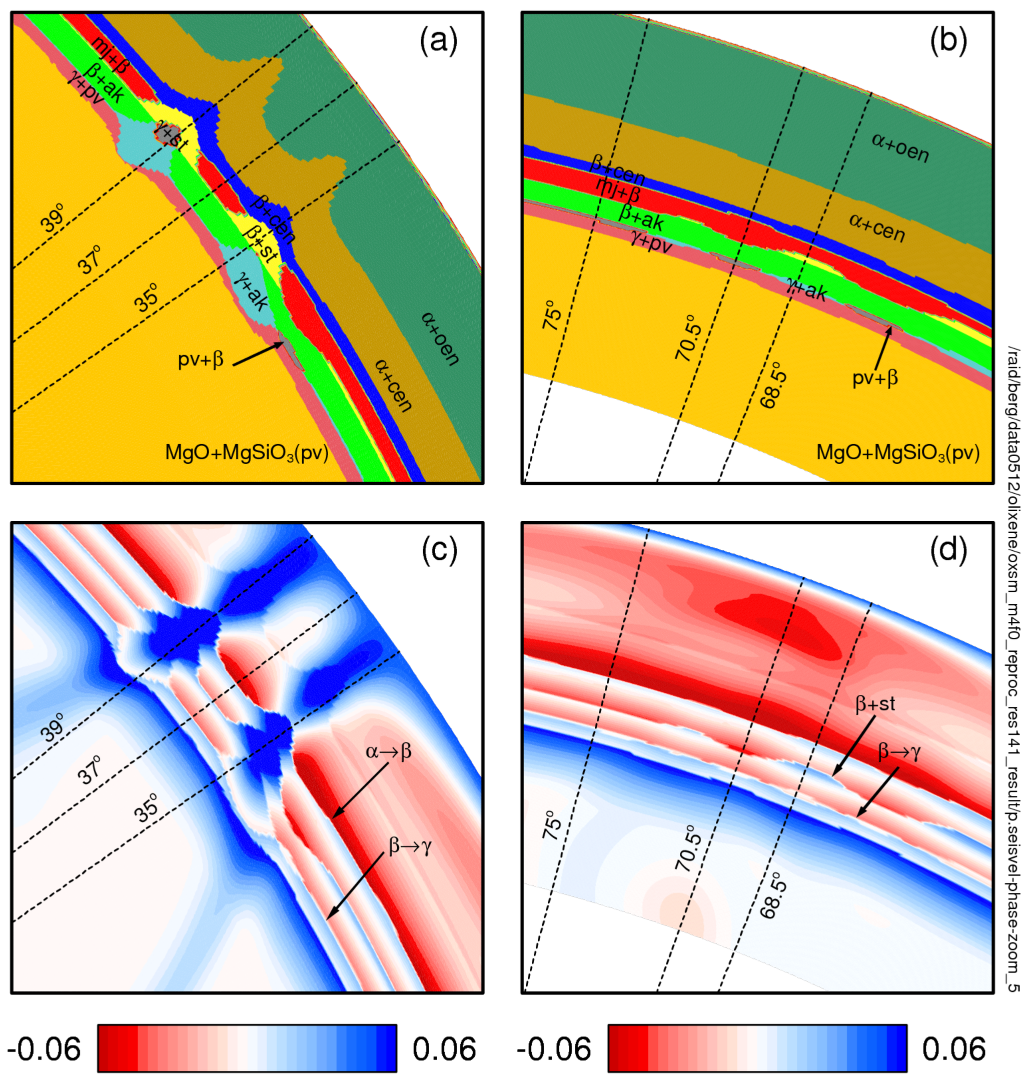
Thomas Geenen [1], Timo Heister [2], Martin Kronbichler [3], Arie van den Berg [1], Michael Jacobs [4], Wolfgang Bangerth [2]

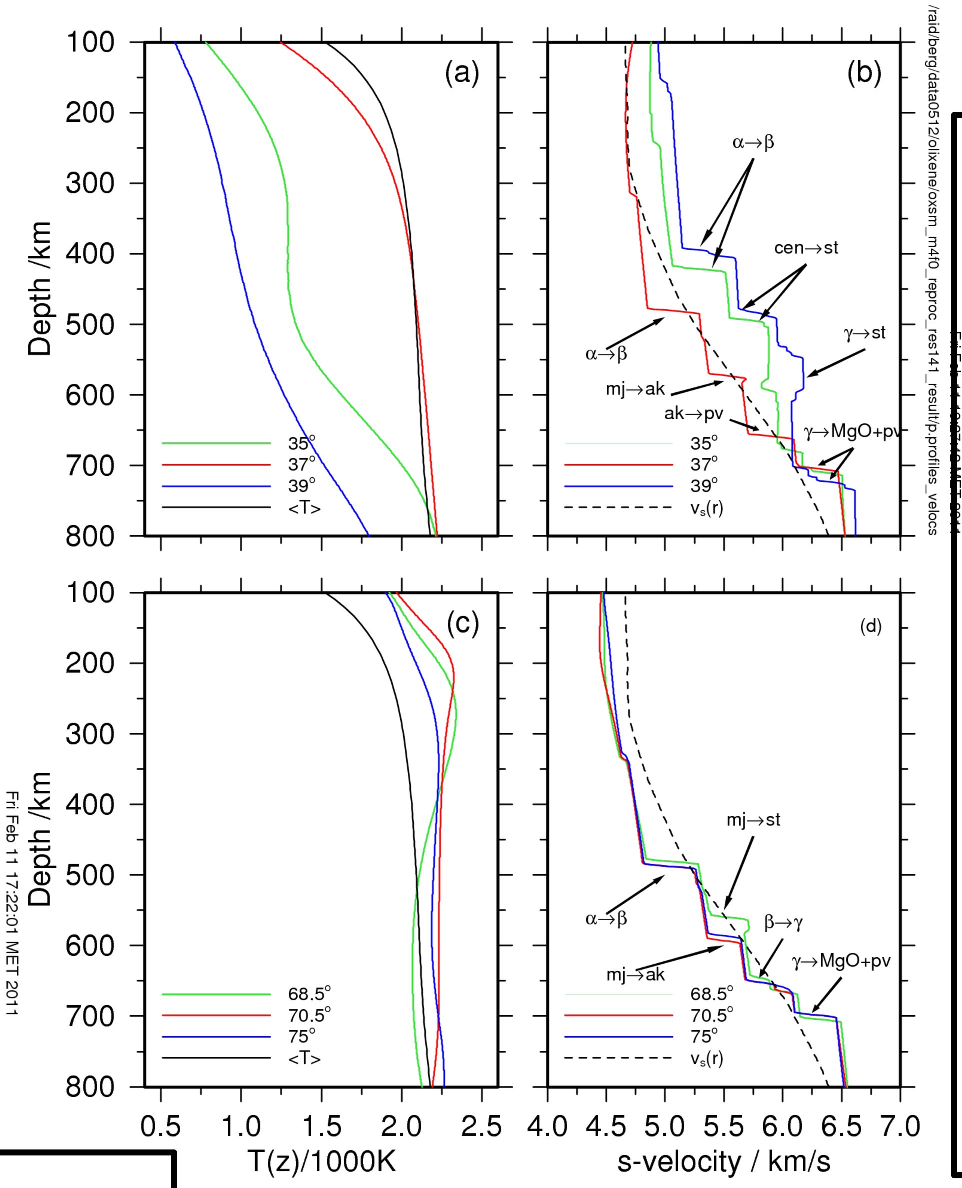
1. Earth Science, University Utrecht, Utrecht, Netherlands.
2. Mathematics, Texas A&M University, College Station, TX, United States.
3. Department of Information Technology, Uppsala University, Uppsala, Sweden.
4. Institut für Metallurgie, Technical University of Clausthal, Clausthal-Zellerfeld, Germany.

geenen@geo.uu.nl

Abstract:
We present high resolution 3D results of the complex mineral phase distribution in the transition zone obtained by numerical modelling of mantle convection.
We extend the work by [Jacobs and van den Berg, 2011] to 3D and illustrate the efficiency of adaptive mesh refinement for capturing the complex spatial distribution and sharp phase transitions as predicted by their model. The underlying thermodynamical model is based on lattice dynamics which allows to predict thermophysical properties and seismic wave speeds for the applied magnesium-endmember olivine-pyroxene mineralogical model. The use of 3D geometry allows more realistic prediction of phase distribution and seismic wave speeds resulting from 3D flow processes involving the Earth's transition zone and more significant comparisons with
interpretations from seismic tomography and seismic reflectivity studies aimed at the transition zone. Model results are generated with a recently developed geodynamics modeling application Aspect (www.dealii.org).
We extended this model to incorporate both a general thermodynamic model, represented by
P,T space tabulated thermophysical properties, and a solution strategy that allows for compressible flow.
When modeling compressible flow in the so called truncated anelastic approximation framework we have to adapt the solver strategy that has been proven by several authors to be highly efficient for incompressible flow to incorporate an extra term in the continuity equation.We present several possible solution strategies and discuss their implication in terms of robustness and computational efficiency.

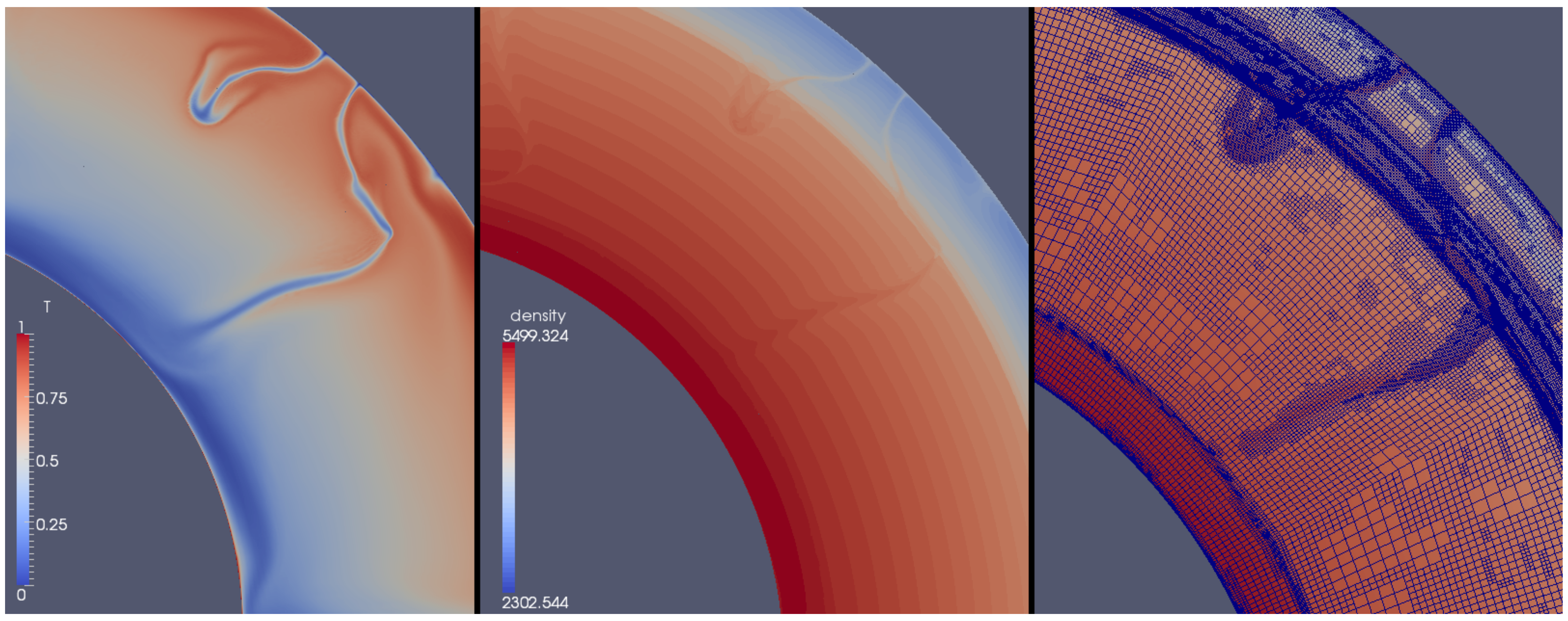






Complex phase distribution in the transition zone

complex spatial distribution of mineral phases in the Earth's upper mantle, result from lateral variations of temperature in a convecting mantle. Our convection model includes a self-consistent thermodynamic description for an olivine–pyroxene composition in the SiO_2, MgO system. The thermodynamic model is based on lattice vibrations and allows for the calculation of thermophysical properties as well as seismic wavespeeds. The figure above illustrates a typical phase profile. The colored lines represent typical crossections through a convective model illustrated in the leftmost figure. The figure on the left illustrates that seemingly smooth temperature profiles can result in a complex phase-depth diagram with corresponding sharp jumps in seismic velocities. Modelling results show a complex structure in the behavior of physical properties, in particular the seismic shear wavespeed, in a depth range including the mantle transition zone, 400–700 km. This behavior is related to the distribution of mineral phases in the olivine–pyroxene system. Especially near cold downwelling flows, representing subducting lithospheric plates, model results show strong lateral variation of mineral phases and associated shear wavespeed. In the Figure below we observe so called slab ponding, where a cold descending slab gets trapped in the transition zone before sinking into the deeper mantle. This results in a broad region with complex phase distributions as shown in the figure on the left. Figures above and on the left used with permission from the authors [1].



Compressible flow solver:

We use the Truncated Anelastic Liguid Approximation (TALA) in our flow model to account for the increase in density with pressure.
This results in a contribution to the continuity equation.

$$\nabla \cdot (\rho_r \mathbf{u}) = 0$$

As a result our system of equation can be written as,

$$\mathbf{A}\begin{bmatrix}\mathbf{u}\\ \mathbf{p}\end{bmatrix} = \begin{bmatrix}\mathbf{Q} & \mathbf{G^T}\\ \mathbf{G} & \mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ \mathbf{p}\end{bmatrix}$$

With **C** the extra term resulting from the TALA formulation

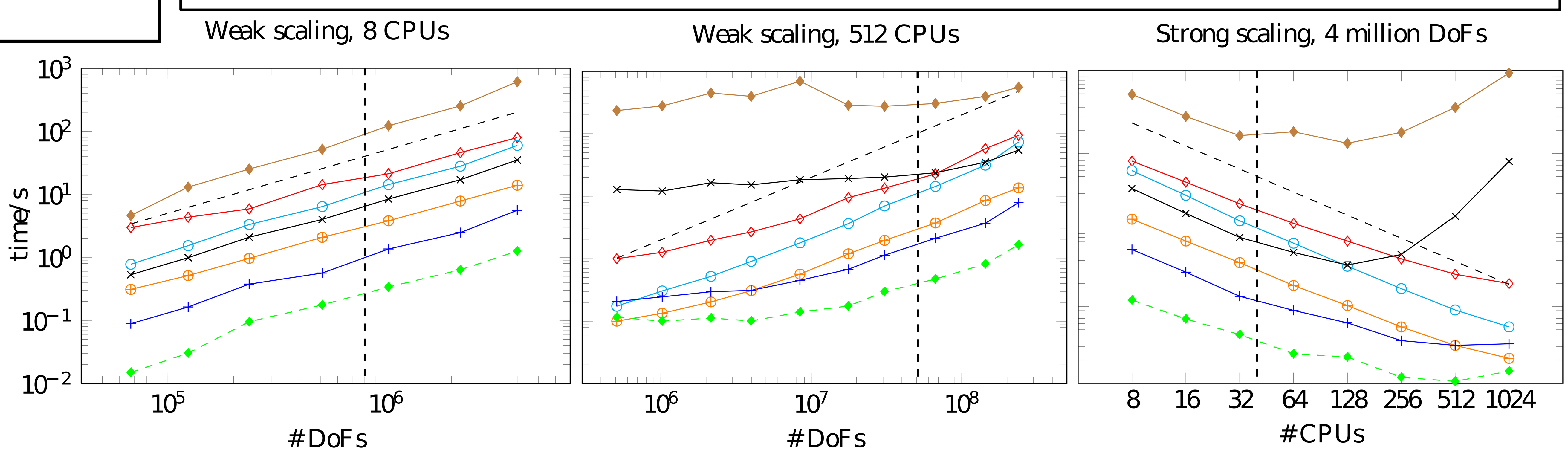
$$\frac{\mathbf{u}}{\rho}\frac{\partial \rho}{\partial z}$$

This renders previously successfully applied preconditioner strategies for the incompressible Stokes equation inefficient, since there is no good approximation to **C** that can be constructed and/or applied cheaply.
We deal with this by moving this extra term **C** to the right hand side and treat its contribution explicitly.
We found that the solver converges as before if we are careful that the average value of this contribution is zero over the whole domain.
The heat equation is augmented with three extra terms from the extended Bousinesq equation, adiabatic heating, viscous dissipation and radiogenic heating, all treated explicitly in the time integration

Scaling of the modelling application:

From the figures below it is apparent that all operations in our program scale well with increasing problem size (weak scaling) once the problem size per MPI process becomes large enough. Likewise, run times can be reduced inversely proportional to the number of processors (strong scaling) as long as the local size or the problem is sufficiently large. The threshold for this scalability is approximately a minimal local problem size of 100,000 degrees of freedom per MPI process, indicated by the vertical lines in the figure below. Figures below used with permission from the authors [2].

Weak and strong scaling experiments for one time step of a 3D mantle convection simulation. In each of the graphs, the vertical line indicates 10e5 degrees of freedom per processor core; cores have more than this threshold to the right of the line in the top two panels, and to the left of the line in the strong scaling results.



Acknowledgements:
part of these results are obtained with the new 3D mantle convection code **Aspect** developed with support from CIG

[1] Jacobs M.H.G., Van den Berg A.P. Complex phase distribution and seismic velocity structure of the transition zone: Convection model predictions for a magnesium-endmember olivine-pyroxene mantle(2011) Physics of the Earth and Planetary Interiors, 186 (1-2), pp. 36-48.

[2] Martin Kronbichler, Timo Heister and Wolfgang Bangerth High Accuracy Mantle Convection Simulation through Modern Numerical Methods, submitted(2011) to Geophysical Journal International

Universiteit Utrecht

ISES

NWO Nederlandse Organisatie voor Wetenschappelijk Onderzoek

Faculty of Geosciences

Earth Sciences