The effect of elastic compressibility in geodynamic models using a benchmark of a sinking cylinder in a bounded medium

Walter Austmann[1], Rob Govers[1]
[1]Department of Earth Sciences, Utrecht University, Utrecht, The Netherlands
E-mail: w.f.austmann@uu.nl

Introduction

Although it is commonly accepted that the Earth is compressible, this effect is neglected in most numerical models. For purely viscous rheologies, this is justified, since the bulk viscosity of the Earth is negligible. However, for visco-elastic rheologies, the elastic deformation allows volumetric changes. To study the effect of elastic compressibility, a well-defined benchmark in fluid mechanics is used: a sinking cylinder in a homogenous fluid. An analytical solution exists in the case of an incompressible purely viscous fluid and is compared to our visco-elastic solution.

Benchmark description

In the analytical benchmark, the steady state Stokes equations are solved for a rigid cylinder moving with a constant velocity through a homogeneous fluid between two parallel impermeable walls. The fluid extends infinitely in the direction of motion. The resulting flow field (Fig. 2) is only non-uniform in a small confined region around the cylinder (~2 radius width).
From the flow field, the dimensionless dragforce is calculated. In our models, it is assumed that when the cylinder is in steady state, the dragforce balances the buoyancy force. This implies that the fluid is not self-gravitating.

Model Setup

A rigid cylinder is placed midway in a homogeneous visco-elastic fluid. The fluid is bounded by impermeable walls. The tank is 25cm high and 1cm wide. The cylinder is placed at a distance of 1,5cm from the top of the domain. A sketch is given in Fig. 1 and the model parameters in Table 1.
This setup is solved using the lagrangian finite element package G-TECTON consisting of both triangular and quadrilateral elements. A remeshing is used to avoid numerical errors due to high deformations.

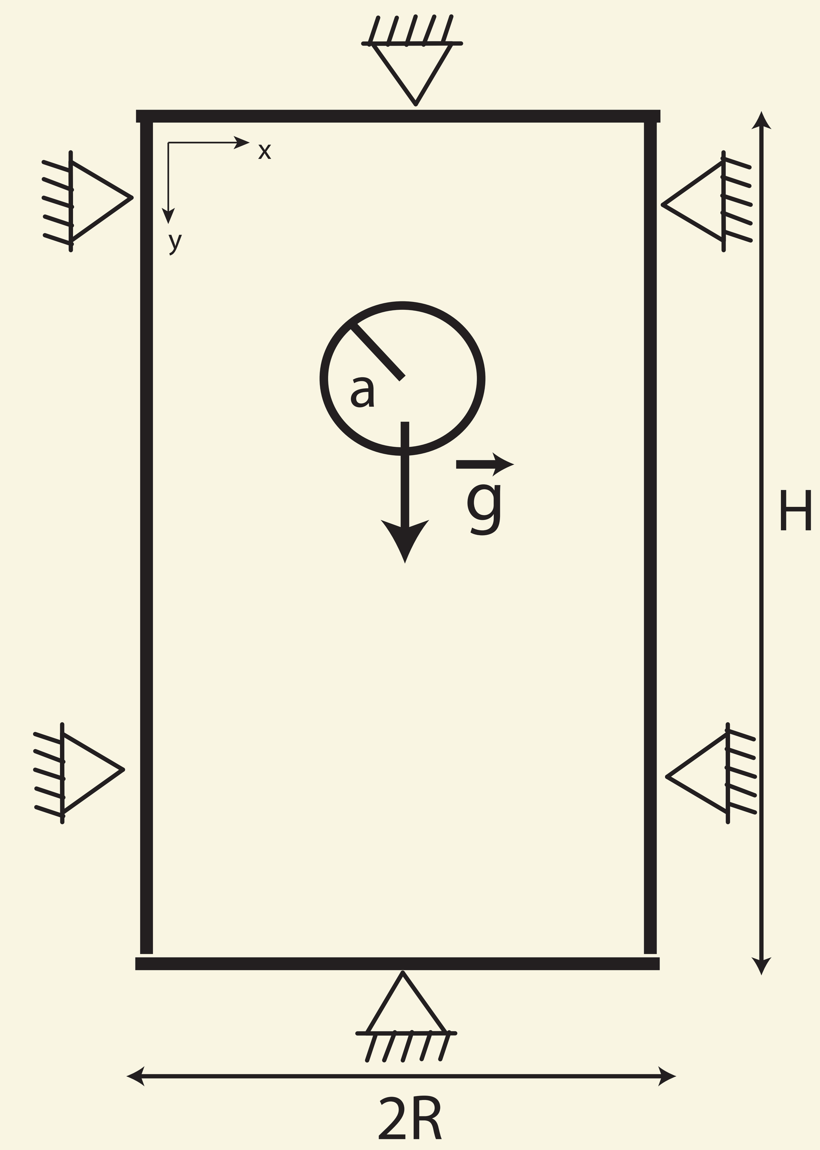

Figure 1: Physical sketch of the model. In the reference model H=25cm; R=0.5cm; a=0.25cm. After:DeFranco (2008)

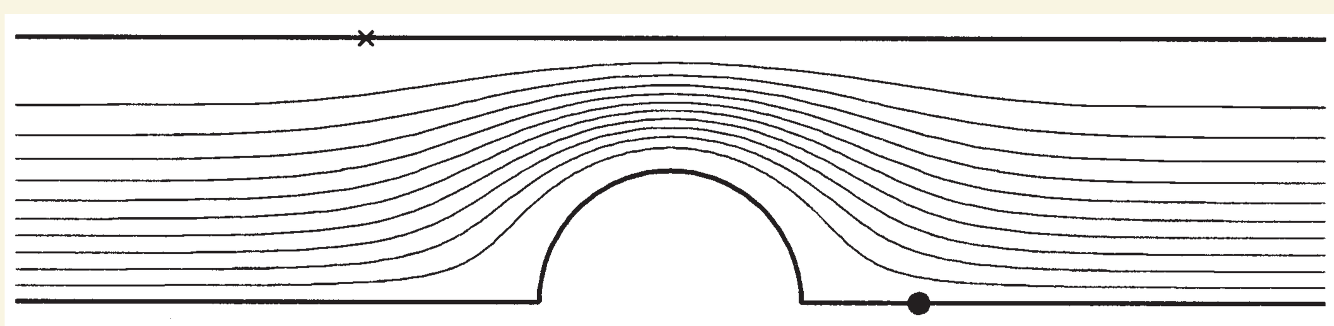

Figure 2: Streamfunction of the benchmark around a fixed cylinder (Liu et. al, 1998)

Material	Cylinder	Fluid
Viscosity (Pa s)	$1.64*10^{20}$	1.64
Young Modulus (Pa)	$3.5*10^{13}$	350
Poisson ratio	0.4	0.4
Density (kg m^{-3})	7161	0

Table 1: model parameter of the reference model

Results

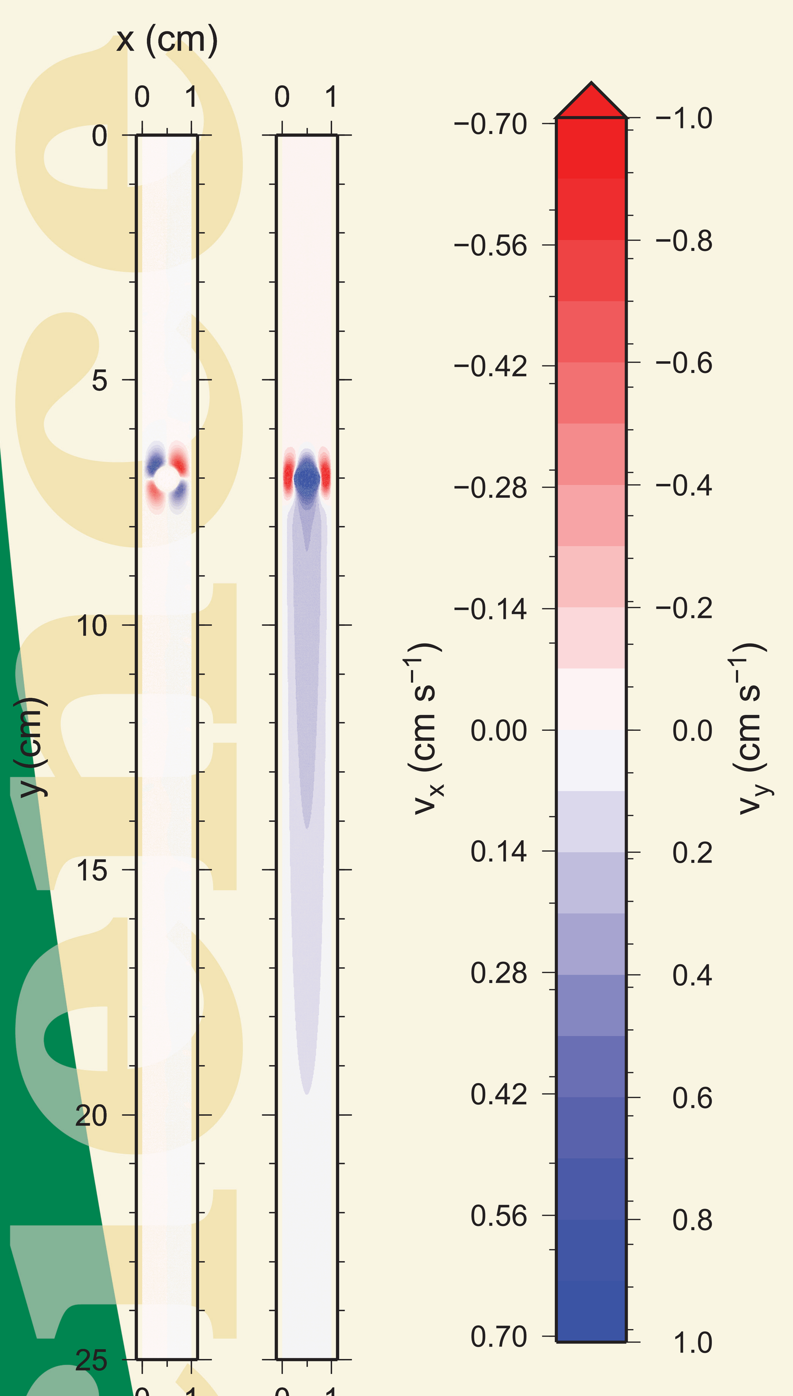

Figure 3: Velocity field after 5 seconds. The cylinder has a velocity close to steady state. Horizontal velocity on the left and vertical velocity on the right. Different scale bars are used for the two components.

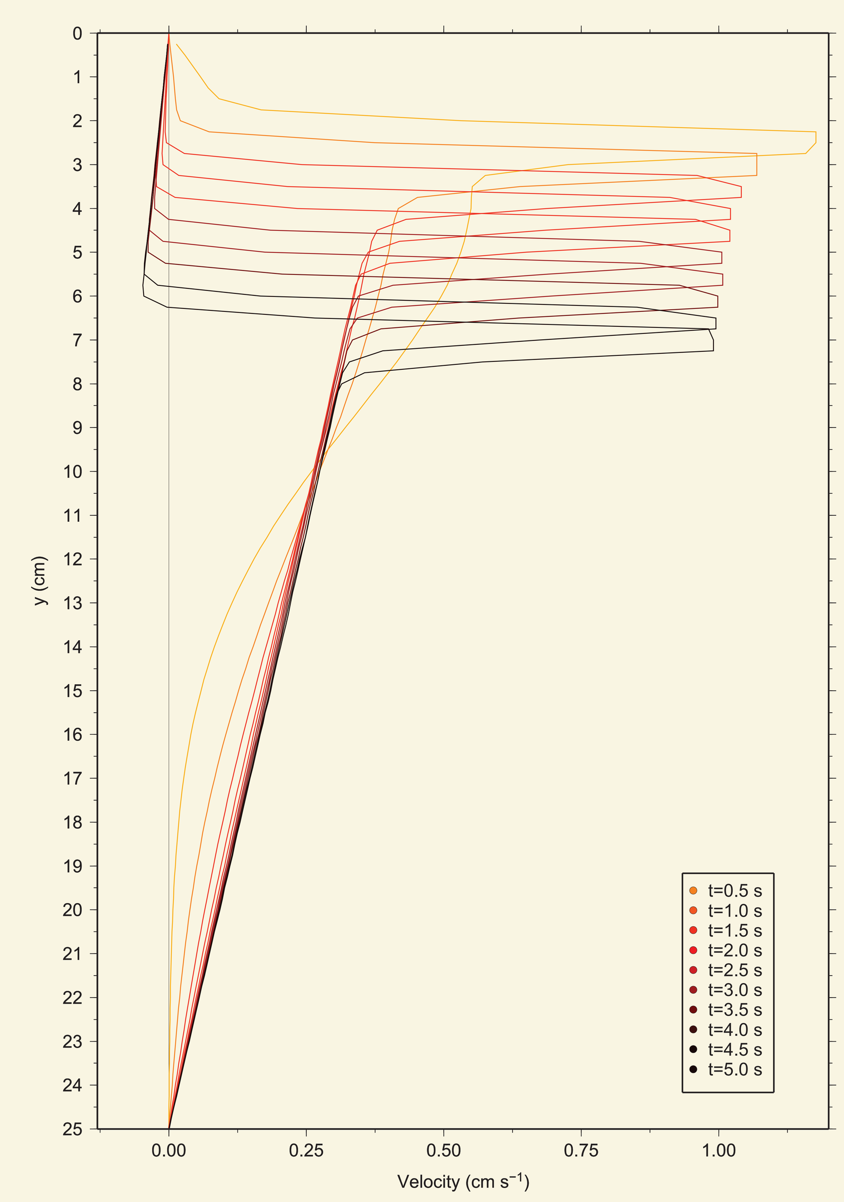

Figure 4: Time evolution of the vertical velocity at x=0.5cm. For near steady state velocities, the vertical motion of the fluid approaches a linear profile. The gradient is dependent on the distance between the cylinder and the domain boundaries. Thus, as long as the distance travelled by the cylinder is much smaller than the distance between the cylinder and the boundary, the cylinder approaches a steady state velocity.

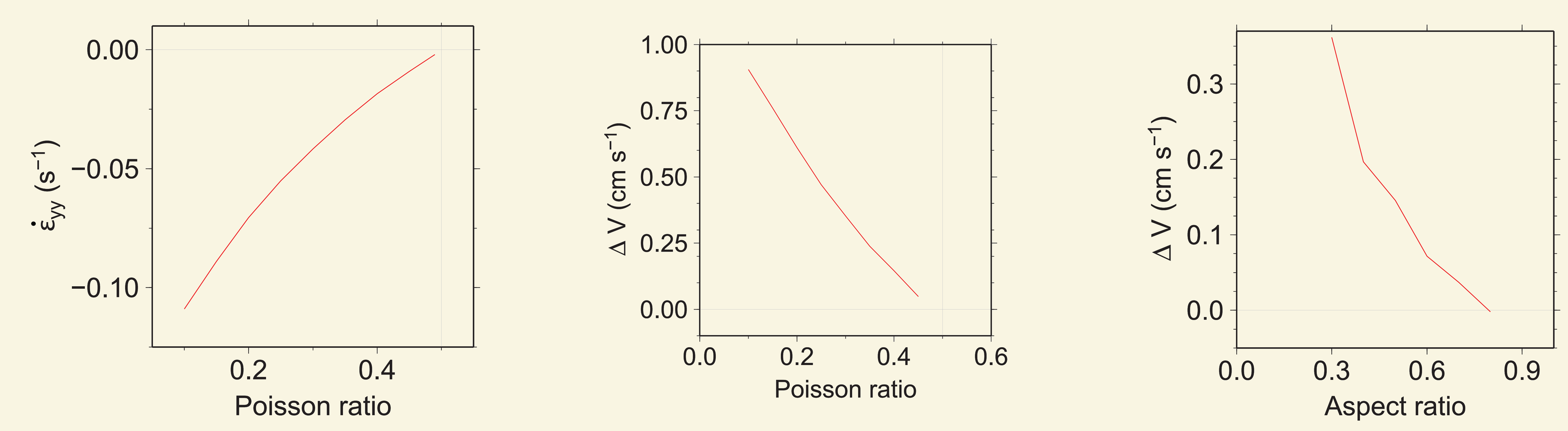

Figure 5: For an increasing Poisson ratio (1) the flow becomes more confined (left: the average vertical strain rate in the region below the cylinder approaches zero) and (2) the vertical velocity of the cylinder approaches the analytical solution (ν=0.5) (middle)
Vertical velocity differences decrease for increasing a/R

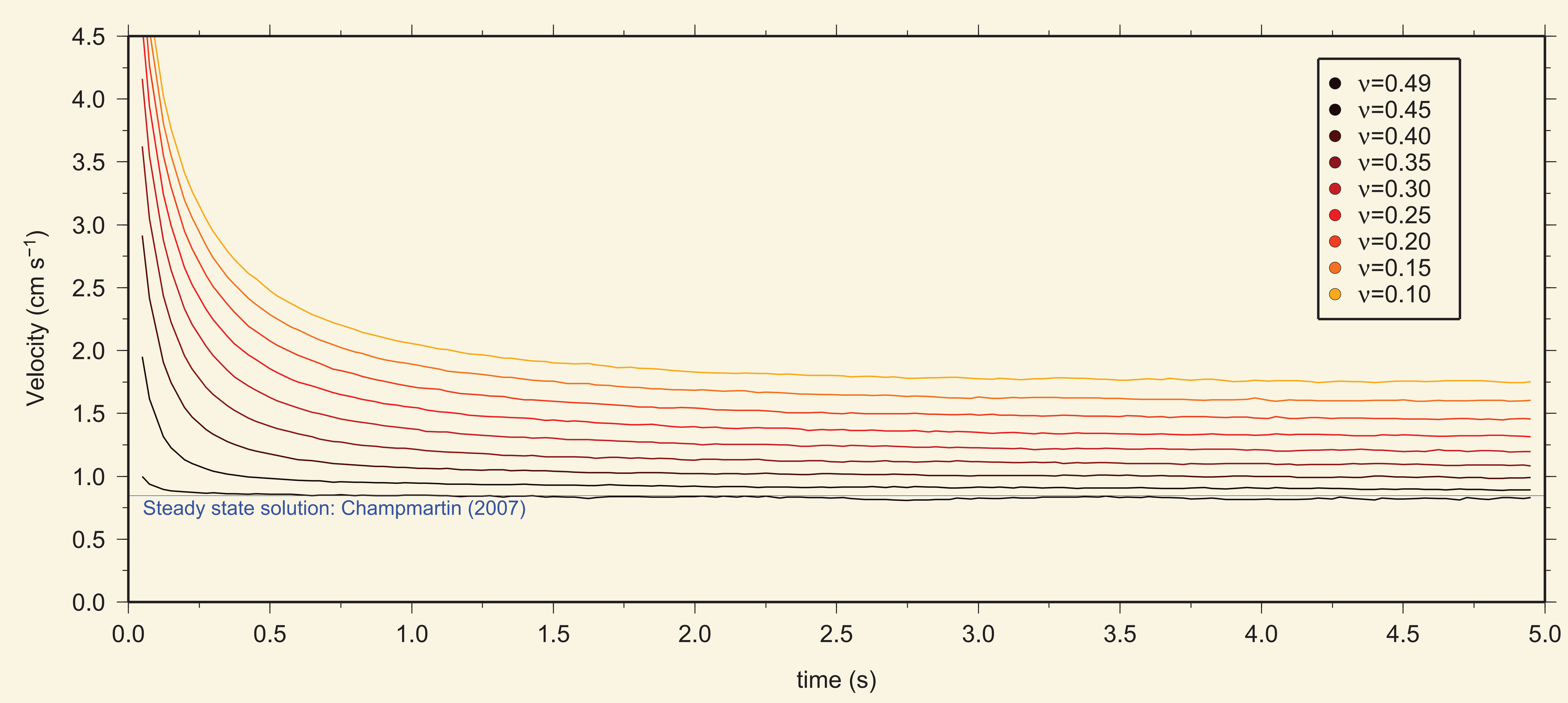

Figure 6: Velocity profiles for different Poisson ratios. A Poisson ratio of 0.5 represents an incompressible flow. Each model run reaches an almost constant velocity at the end of the run. The highest Poisson ratios are in good agreement with the analytical solution. Due to elastic effects, the initial velocity is large and it monotonically decreasing with time.
For Earth-like poisson ratios (i.e. ν=0.3) the difference with the incompressible (analytical) solution is 40%.

Analysis

In the incompressible solution, the downward motion of the cylinder is balanced by the upward flow around the cylinder. In the compressible solution, a part of the motion of the cylinder is accomodated by the compressibility of the fluid below. Thus the upward motion is smaller and the dragforce is decreased. Compressibility increases therefore the sinking velocity of the cylinder.

Universiteit Utrecht

Discussion

The motion of tectonic plates is the effect of driving and resisting forces acting on the plate. The driving forces (e.g. slab pull, ridge push and continental lithospheric body forces) are relatively well constrained. The resisting forces are much harder to quantify and are only implicit in the numerical models. By using an incompressible viscous rheology instead of a (more realistic) compressible visco-elastic rheology, these resistive forces are overestimated. This causes an underestimation of the deformation rates (e.g. predicted plate velocities, sinking velocities of (detached) slabs or topographic uplift). In simple models this can be as large as 40% for Earth-like Poisson ratios.

Conclusions

- Compressible visco-elastic flow increases the steady state velocity by 40% in the case of a sinking cylinder for Earth-like Poisson ratios.
- The motion of the fluid is no longer confined to a small region around the cylinder
- The boundaries of fluid have large effect on the motion: the closer a boundary, the larger the resistance on the motion
- Steady state results are achieved when the cylinder is initially (1) close to the top domain to damp out the visco-elastic overshoot and (2) far from the bottom so that the cylinder is now slowed down after obtaining the steady state velocity.