

# A numerical study of nonlinear wave interactions

OS11A-1238

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## Objective

Nonlinear triad interactions redistribute energy which:

- Transforms the shape of sea-swell waves (SS,  $f = 0.05 - 2$  Hz)
- Generates energy at infragravity frequencies (IG,  $f = 0.005 - 0.05$  Hz)

IG waves are found to be important in the erosion of beaches and dunes during storms. Recently, it has been suggested that IG waves may lose energy by:

- Transferring it back to (former) SS spectral peak
- IG-IG transfers that cause IG waves to steepen and to eventually break

Here, we investigate energy transfer patterns for different types of beaches, using the model SWASH.

## Variable bathymetries

We use SWASH to simulate wave propagation over a low (1/80), mild (1/50) and steep (1/20) sloping beach, and a 1/80 beach with a sandbar, with  $H_s = 0.1$  m, and  $T_p = 2.25$  s.

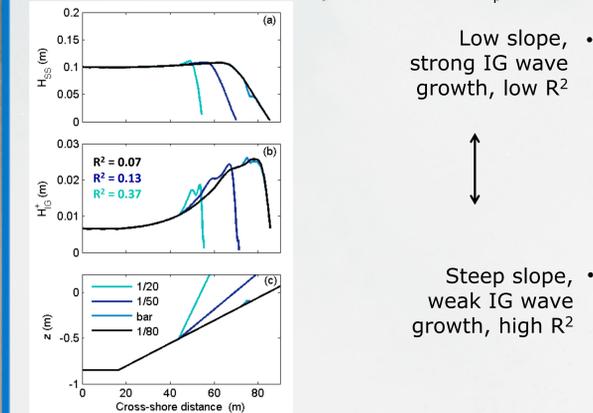
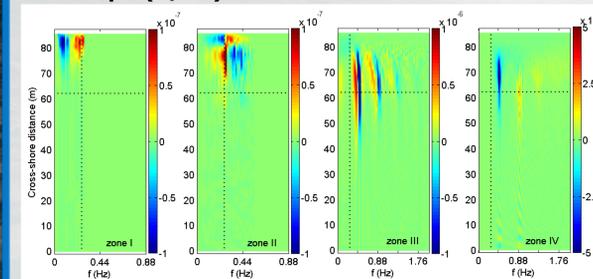


Figure 2: (a)  $H_{ss}$  and (b)  $H_{IG}$ . Panel (c) shows corresponding bottom profiles. Reflection  $R^2$  in inner surf ( $h = 5$  cm). Sand bar dimensions correspond to  $\Delta h = 70$  cm,  $\Delta x = 80$  m,  $h = 1.80$  m in the field.

## Energy transfers

The nonlinear source term  $S_{nl}$  accounts for energy transfers to and from a frequency  $f$ .  $S_{nl}$  is estimated by integrating the product of the imaginary part of the bispectrum and a coupling coefficient following Herbers et al. 2000. Figure 5 shows the  $S_{nl}$  term for each of the four bispectral zones as defined in the box 'Bispectral analysis', for the 1/20 and 1/80 slopes.

### Low slope (1/80)



### Steep slope (1/20)

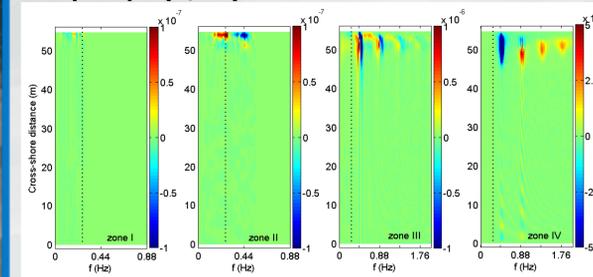


Figure 5:  $S_{nl}$  terms plotted versus  $f$  and  $x$ . Vertical dashed line is the cut-off between IG and SS frequencies. Horizontal dashed line is the edge of the surf zone.

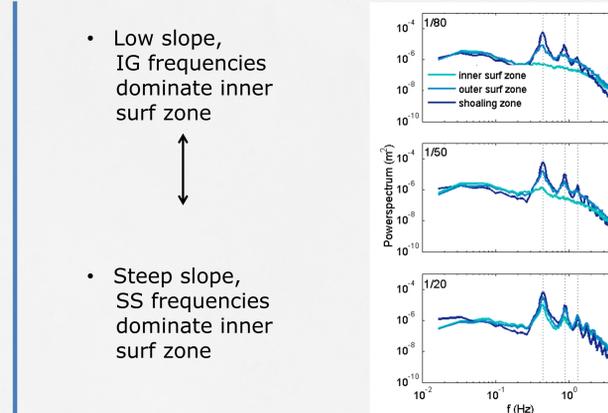


Figure 3: Powerspectra in shoaling ( $h = 25$  cm), outer surf ( $h = 12.5$  cm) and inner surf zone ( $h = 5$  cm) for the three beach types. Vertical lines indicate offshore power spectral peak (0.44 Hz) and its first (0.88 Hz) and second (1.32 Hz) harmonic.

## Inner surf zone

### Low slope

- Transfers involving two or more IG frequencies dominate (I,II)
- Energy cascades from low to high IG frequencies and 'harmonics' (I,II,III)

### Steep slope

- Transfers involving two or more SS frequencies dominate (III, IV)
- IG interactions are weak, small transfer/loss

## Sandbar

At bar locally stronger transfers, shoreward of the bar weaker transfers

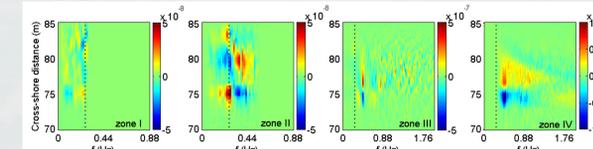


Figure 6: Difference in  $S_{nl}$  of bar minus non-barred 1/80 slope, plotted versus  $f$  and  $x$ .

## Bispectral analysis

The bispectrum  $B_{f_1, f_2}$  detects phase-coupling between frequency components in a triad. The discrete bispectrum is defined as,

$$B_{f_1, f_2} = E[A_{f_1} A_{f_2} A_{f_1 + f_2}^*]$$

$E[\ ]$  ensemble average  
 $A$  complex Fourier coefficients  
 $*$  complex conjugation

To obtain insight in the different triad interactions, four zones are defined, for which nonlinear energy transfers are calculated.

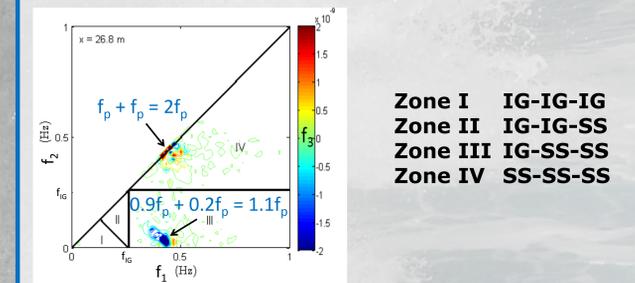
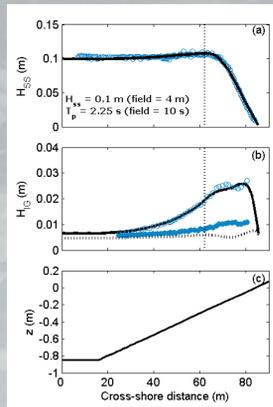


Figure 4: The imaginary part of the bispectrum at  $x = 26.8$  m.

## Model validation

Governing equations of SWASH are the non-linear shallow water equations and account for non-hydrostatic pressure (Zijlema et al. 2011). We validated (Figure 1) SWASH using the high-resolution, small-scale, Globex lab dataset with a 1:80 sloping beach (Ruessink et al. 2013).

- Onset of SS wave breaking and dissipation amount captured well



- IG wave height increase and arrest in good agreement

- IG dissipation slightly overestimated

Figure 1: Measured (dots) and modelled (lines) significant wave height  $H_s$  versus cross-shore distance  $x$ , for (a) SS waves and (b) incoming (circles) and outgoing (dots) IG waves.

## Conclusions

- Low slope enhances IG wave growth. IG interactions dominate in shallow water, resulting in IG energy loss.
- Steep slope limits IG wave growth and thereby IG interactions, resulting in less IG energy loss.
- At a sandbar locally stronger transfers, this is compensated by weaker transfers shoreward of the bar.

## Outlook

- Other wave conditions
- Other bar dimensions, concave/convex beach shapes