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# Beyond Turing: The response of patterned ecosystems to environmental change



### Introduction

Currently, mathematical analysis of spatially explicit ecological models relies mainly on Turing analysis. Recently, mathematical techniques that go beyond Turing analysis have been applied to ecological models. Here we show how stability analysis on patterned system states helps in understanding the behavior of patterned ecosystems subject to environmental change.

## **Turing analysis and beyond**

#### **Turing prediction region**

The stability of ecosystem states can be determined by adding perturbations

#### Turing stability analysis:

- considers stability of uniform system states
- assumes perturbations to be sinusoidal

#### Turing prediction region:

 here patterns can be expected, assuming that patterns adopt perturbation wavenumber when they form

## **Environmental Change**

#### Rules of adaptation:

- 1. If system is in the Busse balloon: patch biomass changes
- 2. When system is forced outside the Busse balloon: pattern wavenumber changes

## Stability regions for an arid ecosystem model

no vegetation patterned vegetation

tion uniform vegetation









If rainfall declines, the system leaves the Busse balloon and first encounters the sideband instability. Here patches go extinct, resulting in a lower wavenumber.





perturbation wavenumber

Perturbations with a positive growth rate trigger pattern formation

#### **Busse balloon**

Busse balloon: region with stable patterns.

Notice that:

- Multiple stable patterned states exist.
- Pattern wavenumber decreases with rainfall.
- Only stable patterns can be observed, so:
  - patterns can be observed outside
  - Turing prediction region.
  - not all patterns in Turing prediction



Stability regions. The wavenumber is inversely related to the space between patches.

floquet wavenumber

Perturbations with a positive growth rate destabilize a pattern

**Sideband instability** 

W. Eckhaus (1930-2000)



**One-by-one extinction of patches** 

Period doubling instability

NEW

If change in rainfall is rapid, the system also crosses the period doubling instability. Here half or all patches go extinct simultaneously.

## region can be observed.



A.M. Turing

(1912-1954)

NEW

## Conclusion

Knowledge about the stability and destabilization of patterned ecosystem states is essential for understanding the response of patterned ecosystems to environmental change.



#### References

- Busse, F., 1978. Non-linear properties of thermal convection. Reports on Progress in Physics, 41, 1929-1967.
- Siteur, K., E. Siero, M.B. Eppinga, J.D.M. Rademacher, A. Doelman & M. Rietkerk, 2014. Beyond Turing: The response of patterned ecosystems to environmental change. Ecological Complexity, 20, 81-96.
- Turing, A. M., 1953. The chemical basis of morphogenesis. Bulletin of mathematical biology, 237, 37-72.

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