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Faculty of Geosciences
Environmental Sciences

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To $b=1$ or not to $b=1$. Numerical, conceptual, hydraulic and geometric explanations for observed streamflow recession behaviour — a case of being right for which reason?

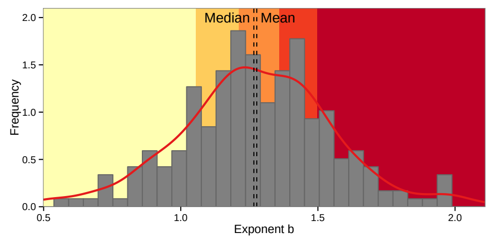
1 Observed streamflow recession

A key question in hillslope and catchment hydrology is how empirical values for recession exponent b as found by the top-down Brutsaert-Nieber streamflow recession analysis

$$-dQ/dt = aQ^b \tag{1}$$

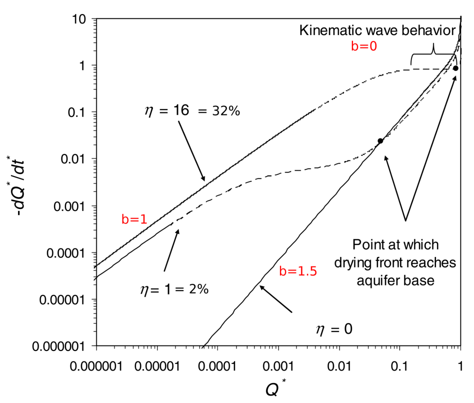
can be explained from underlying bottom-up physical theory such as the Boussinesq equation

$$\frac{\partial h}{\partial t} = \frac{k}{f}\frac{\partial}{\partial x}\left[h\left(\frac{\partial h}{\partial x}\cos\alpha + \sin\alpha\right)\right] + \frac{N}{f}. \tag{2}$$

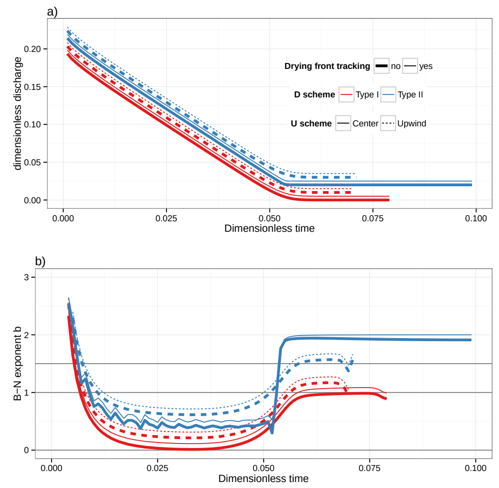


Empirical data (here: 220 catchments in Sweden) suggest that exponent b varies mostly within the range 1–1.5.

2 Numerical explanations



Naïve interpretation of the recession response from a Boussinesq model applied to sloping aquifers could lead to a conclusion of $b=1$ during Late-time. Adapted from *Rupp and Selker*, [2006]



A correct interpretation of Late-time $b=0$ renders the straightforward nonlinear Boussinesq equation inconsistent with observations. From *Bogaart et al.*, [2013].

Conclusion: $b \geq 1$ from the nonlinear Boussinesq equation applied to sloping aquifers is likely due to naïve interpretation of numerical model output.

2.1 Conceptual background

From Equation (1) it follows that

$$Q(t) = \begin{cases} c_1 e^{-at} & \text{for } b=1 \\ [(b-1)(at+c_2)]^{\frac{1}{1-b}} & \text{for } b \neq 1 \end{cases} \tag{3}$$

and three cases can be distinguished:

$b<1$: a finite volume V is drained in finite time $t = -c_2/a$.
$1 \leq b < 2$: a finite volume V is drained in *infinite* time.
$b \geq 2$: an *infinite* volume V is drained in infinite time.

Conclusion: Numerical implementations of Eqn (2) often don't drain *completely*, and therefore shift towards artificial $b=1$ behaviour.

Summary

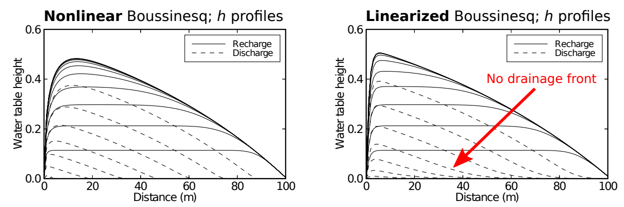
We conclude that explanations of observed $b=1$ based on the linearized Boussinesq equation applied to sloping aquifers are probably flawed. We suggest that observed $b=1$ to 2 from these aquifers is more likely to be due to system properties like conductivity decreasing with depth and and divergent planform, which both have a positive effect on b.

2.2 Linearization

Equation (2) is often linearized by replacing a dynamic h by a constant pD:

$$q = -kh\left(\frac{dh}{dx}\cos\alpha + \sin\alpha\right) \implies -kpD\frac{dh}{dx}\cos\alpha - kh\sin\alpha \tag{4}$$

which can be shown to lead to $b=1$ [*Brutsaert and Nieber*, 1977].



Numerical implementations of Eqn (4) demonstrate that water 'sticks' to the bedrock surface, preventing drainage in finite time, c.f. *Stagnitti et al.*, [2004].

Conclusion: The linearized Boussinesq equation's $b=1$ behaviour is consistent with the same mechanism plaguing numerical implementations of the nonlinear Boussinesq equation.

3 Hydraulic explanations

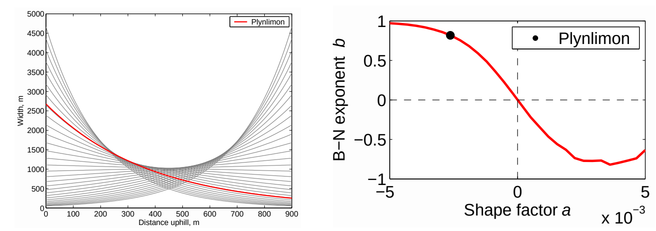
The straightforward assumption of uniform k in (2) can be relaxed, e.g by assuming a power-law profile $k = k_D(z/D)^n$ such that

$$b = \frac{2n+1}{n+1} \tag{5}$$

Although the uniform-k case $n=0 \implies b=1$ again should be considered an artifact, (5) enables $b=0$ to 2. Similarly, TOPmodel's assumption of exponential k-profile leads to $b=2$.

Conclusion: Non-homogeneous soils in conjunction with the Boussinesq or Kinematic Wave equation *do* provide explanations of $b \geq 1$.

4 Geometric explanations



Most constant-k applications of the Boussinesq eqn assume unit-width hillslope geometry, resulting in $b=0$. If a more representative divergent geometry is assumed [*Bogaart and Troch*, 2006], it can be shown that $b \to 1$.

Conclusion: Planform geometry is a significant factor in explaining observed b values

References: Bogaart and Troch (2006), Curvature distribution within hillslopes and catchments and its effect on the hydrological response, *Hydrol. Earth Syst. Sc., 10*. **Bogaart et al. (2013)**, Late-time drainage from a sloping Boussinesq aquifer, *Water Resour. Res., 49*. **Brutsaert and Nieber (1977)**, Regionalized drought flow hydrographs from a mature glaciated plateau, *Water Resour. Res., 13*. **Rupp and Selker (2006)**, On the use of the Boussinesq equation for interpreting recession hydrographs from sloping aquifers, *Water Resour. Res., 42*. **Stagnitti et al. (2004)**, Drying front in a sloping aquifer: Nonlinear effects, *Water Resour. Res., 40*.