Methods for joint inversion of waveform and gravity information for 3D density structure



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Linking classical seismic tomography and mantle dynamics is difficult because seismic traveltimes are mainly sensitive to wave speeds rather than density.

Density variations drive convection and serve to discriminate between thermal and compositional heterogeneities inside the Earth. However, classical seismological observables and gravity provide only weak *constraints with strong trade-offs.*

We show that waveform inversion in alternative parametrisations, using gravity measurements and a fixed v_s, v_p model, results in significant density recovery in synthetic models on the scale of Earth's mantle.

Inversion tests

We run inversions in a roughly **Earth mantle-like 2D** *domain*, with PREM as background model, receivers at the surface and sources at reasonable depth:

- 1. *Reference inversion*,
- in ρ - μ - λ parametrisation
- using seismic and gravity data
- a fixed velocity model

These results are compared to

- 2. same as (1), but with v_p and v_s unconstrained.
- 3. same as (1), but in ρ - v_s - v_p parametrisation
- 4. same as (1), but using only seismic information.

We use a *multi-scale* approach to avoid cycle-skipping. This means that the lowest frequencies (= the largest scale structure) are inverted for first. We start at frequencies of 0.0067 Hz and increase to 0.0147 Hz - higher frequencies are not necessary for the smooth structures investigated here.

1. Methods - inversion scheme

Inversion is run in $\rho - \mu - \lambda$ or $\rho - v_s - v_p$ parametrisation. While the former has improved sensitivity to density, tradeoffs to velocity structre are greatly increased. To reduce these, a known v_s, v_p model (e.g. from traveltime inversion) can be kept fixed using constrained optimisation (see 'Constrained optimisation' \rightarrow).

Waveform inversion is carried out with steepest descent gradient minimisation using the normalised L^2 norm of v(t) and g:

 $\prod [misfit_a(1)]$

 $\int_{t=0}^{\infty} \|v_{rec} - v_{obs}\|^2 dt$



 $\sum \frac{\|g_{rec} - g_{obs}\|^2}{\|g_{rec} - g_{obs}\|^2}$ Inversion scheme, intended to optimise density recovery. A starting model is taken where v_s and v_p structure are assumed to be known. Adjoint kernels are calculated based on the seismic part of the misfit functional, gravity kernels are calculated based on gravity misfit. While in the reference case kernels are applied in ρ - μ - λ paramatrisation, the model update is projected onto fixed v_{s} - v_{p} values in ρ - v_{s} - v_{p} parametrisation.

2. Results



True density model. On top of a 2D 'cartesian' slice of Earth mantle, Gaussian positive and negative density anomalies of 1000 kg/m³ are added on top of background model PREM.

1. Inversion result after 100 iterations using an inversion scheme where seismic and gravity information is used, vs and vp are kept at PREM values and inversion is done in $\rho-\mu-\lambda$ parametrisation.



and vp occur.



2. Inversion result when velocities are not kept 3. Inversion result when the inversion parametri- 4. Inversion result when no gravity information fixed at PREM values. Enormous trade-offs to vs sation is p-v_-v_ . This results in focusing at the is used. This results in focusing at the top of the bottom of the domain that worsens as iterations domain that worsens as iterations progress progress



A constrained optimisation approach allows us to fix certain parameters. The solution of each iteration is projected onto the model space that satisfies the constraints. In the inversions presented here, v_s and v_n serve as constraints, but also e.g. the total model mass can be fixed.



Cartoon of how constrained optimisation works. Misfit varies along all axes, but if v_s,v_p, and/or total mass are fixed, the update is projected back onto that plane.

3. Conclusions & outlook

We propose an inversion scheme that is capable of recovering density using waveform and gravity information, alternative parametrisations, constrained optimisation and a multi-scale approach. Our results indicate that:

This work is intended to be a step towards density recovery in the real, 3D Earth. Continuing in this process, we will next:

- et al. 2013).



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Constrained optimisation

v_p, v_s constant

• density can be recovered in 2D synthetic tests on the scale of Earth's mantle,

• fixing the velocity models aids to counteract tradeoffs of density structure to v_s and $v_{p'}$

 including gravity information helps to avoid focusing effects near receivers,

• inverting in ρ - μ - λ parametrisation helps to avoid focusing effects at depth.

• explore influence of errors in v_s and v_p starting models, as well as the sensitivity to noise,

• assess the use of different types of misfit functionals, e.g. instantaneous phase misfit (Rickers