The imprint of crustal density heterogeneities on seismic wave propagation

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1. Abstract and motivation

- Lateral density variations are the source of mass transport in the Earth at all scales;
- Seismic traveltimes and gravity provide only weak constraints with strong trade-offs and so the density structure of the Earth remains largely unknown.

Traveltimes of body and surface waves do not see density structure due to [1]:
- backward scattering off density perturbations of the body waves in the (p, vs, vp) parametrization
- oscillatory shape of density sensitivity kernels for Rayleigh waves
- much higher sensitivity to S velocity structure for Love waves

We propose to develop a seismic tomography technique that directly inverts for density, using complete seismograms rather than arrival times of certain waves only. The first task in this challenge is to systematically study the imprints of density on synthetic seismograms. To compute the full seismic wavefield in a 3D heterogeneous medium without making significant approximations, we use numerical wave propagation based on a spectral- element discretization of the seismic wave equation.
We compare the imprint of 3D velocity and 3D density structure in the crust with the imprint of the 3D velocity structure on the observed seismograms. The 3D heterogeneities used in simulations are generated randomly and the experiment was performed for a set of different lateral and vertical correlation lengths. We then quantify the possible bias in Q and velocity estimates that may be caused by 3D density structure.

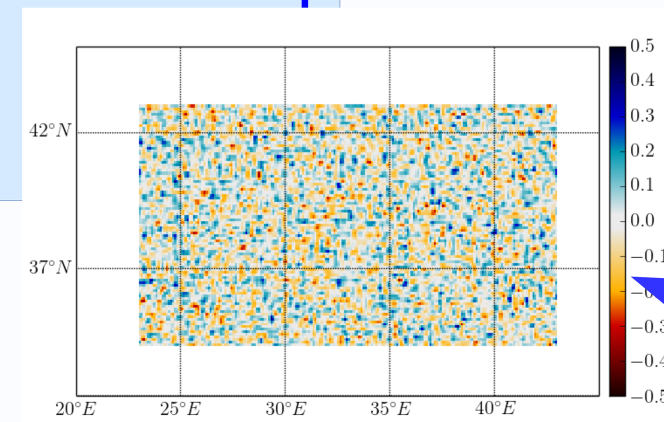
2. The experimental setup

2.1 Random media generation

We generate 3D random media by computing a random (white) phase spectrum, modulating it, using the Fourier transform to obtain space domain representation and then scaling to the desired root mean square value [2]. Lateral correlation lengths used in the simulations presented are: 50 km (complex medium, Figure 1), 200 km (the reference experiment, Figure 2) and 1000 km (smooth medium). Vertical correlation lengths used are, respectively: 10 km, 20 km and 100 km. Each 3D random medium can be used by the numerical package as 3D density, SV, SH or P velocity structure. The 3D structures are superimposed onto the uppermost 40 km of the 1D PREM model [3] with 40 km crustal thickness.

The root mean square of generated heterogeneities is computed using a 3D S wave velocity model of the Anatolia region. We then used scaling velocity - density relations from [4] to estimate the root mean squares of P wave velocity and density variations in the upper crust.

The results are as follows:
- 3D density variations vary approximately 2.2 – 2.9 kg/m^3 peak to peak after superimposing the 3D medium onto PREM (the exact values may vary between different random realizations)
- S velocity variations: 2.2 – 4.2 km/s
- P velocity variations: 3.9 – 7.6 km/s



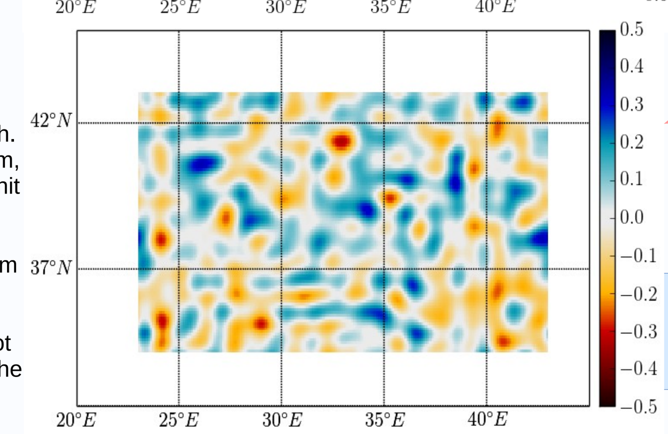


Figure 1 (up): a slice of 3D random medium at 20 km depth. Lateral correlation length: 50 km, unit in colorbar: the structure unit (kg/m^3 for density, km/s for velocity). Those structures are superimposed as variations from the underlying 1D model.

Figure 2 (below): analogical plot for 200 km correlation length (the "reference medium")

2.2 Numerical wave propagation

Mathematical background	Computational grid	Simulations
We simulate elastic wave propagation in heterogeneous media using spectral-elements in a spherical section. It solves the elastic wave equation: $$\rho(x)\,\frac{\partial^2}{\partial t^2}u(x,t) - \nabla\,\sigma(x,t) = f(x,t)$$ Where ρ denotes density, u – the displacement field, f – the external force density and σ – the stress tensor.	- regional scale - 34° to 43° latitude, 23° to 43° longitude (2000 km by 1000 km wide) - 471 km depth to the surface of the Earth - the background 1D model is PREM with 40 km crust - 3D heterogeneities are added only to the uppermost 40 km of the grid	- We calculate 700 s seismograms for 960 receivers distributed regularly on the computational grid (at the surface) - source mechanism: strike-slip (Figure 3) - We compare data for 3D velocity and density structure with data for the same 3D velocity, but 1D density structure - We perform 1 simulation for media of 1000 and 50 km lateral correlation length (smooth and complex media), 5 simulations for 200 km correlation length (different random realizations of the "reference medium")

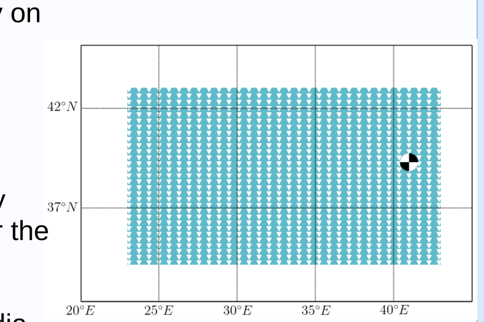


Figure 3: The surface of the computational grid with the virtual receivers (white triangles) and the event (beachball plot)

3. The misfit criteria

We compute time- and frequency-dependent variations in amplitude and traveltimes. This is done as follows:

The traces are first tapered and bandpass filtered in three frequency bands (0.02 to 0.125 Hz, 0.02 to 0.067 Hz and 0.02 to 0.04 Hz).Then we march through the time steps and independently compute time shifts and amplitude differences (see Figure 4 and 5).

In each timestep:

- the filtered trace is multiplied by a Gaussian window with standard deviation different for each frequency band (wider window for lower frequency);

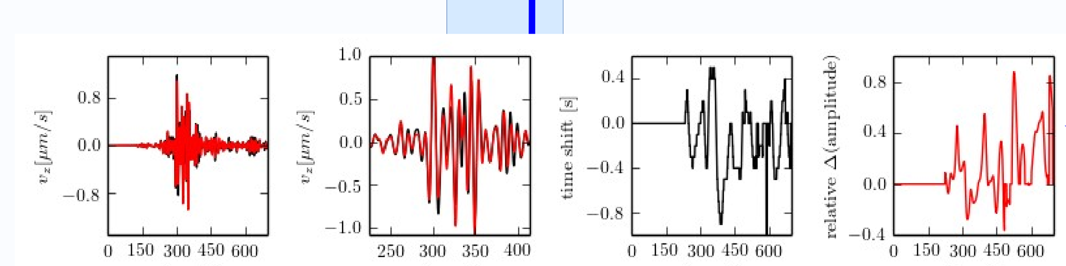
- the time shift is computed as the maximum value of the cross-correlation function:

$$t_{delay} = \underset{\tau}{argmax}\left(\int u(t)u_{ref}(t+\tau)dt\right)$$

where u represents the seismogram for a medium with 3D velocity and 3D density structure, u_{ref} – the reference seismogram for the same 3D velocity, but 1D density structure, and denotes the shift between compared signals

- the relative amplitude difference is computed as:

$$\delta(amplitude) = \frac{\int u^2 dt - \int u_{ref}^2 dt}{\int u_{ref}^2 dt}$$



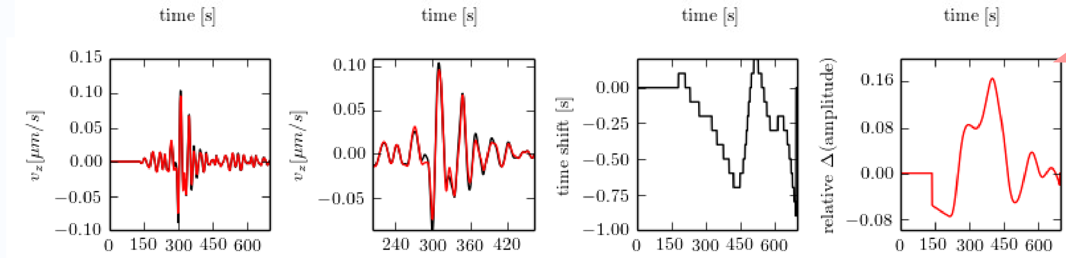


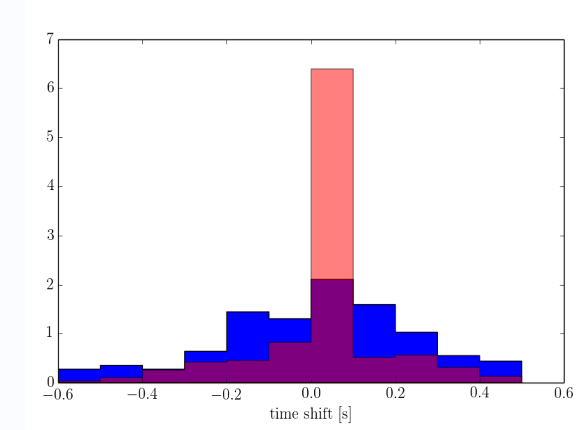
Figure 4 (up): a seismogram (z component) for a chosen station for the medium complexity structure experiment (200 km correlation length), black – 3D velocity and 3D density structure („3D-all"), red – same 3D velocity, but 1D density structure („3D-vel"). Left to right: full seismograms, zoom-in, time shifts between „3D-all" and "3Dvel" calculated by windowed cross-corelation, relative amplitude differences between the two. Frequency band: 0.02 – 0.125 Hz).

Figure 5 (below): analogical plot for frequency band 0.02 – 0.04 Hz

4. Results

We compare seismograms computed for 3D velocity and density („3D-all") structure in the uppermost 40 km with seismograms computed for the same 3D velocity structure, but 1D density kept as PREM with 40 km crust („3D-vel" structure). **All timeshifts and amplitude differences presented in panel 4 are comparisons between the „3D-all" and „3D-vel" structures**

4.1. The effect of medium complexity



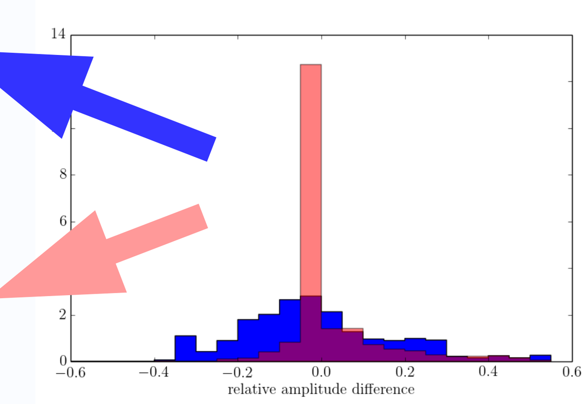


Figure 6 (left): Normed histograms of time shifts stacked for all the stations of the grid for the first 300 s after the first difference between the waveforms. Blue: time shifts for one experiment with 50 km lateral correlation length of the medium (complex medium), magenta: time shifts for five experiments with 200 km lateral correlation length of the medium (reference medium).

Figure 7 (down): analogical plot for amplitude differences

We clearly observe that misfit values grow with growing complexity of the medium.

Conclusion: the more scatterers in the medium, the more significant the density imprint

Table 1: Time shift and relative amplitude difference standard deviations (averaged over components) for experiments with different medium complexity

lateral correlation length	time shift standard deviation	amplitude standard deviation
50 km	0.95 s	0.20
200 km	0.19 s	0.15
1000 km	0.01 s	0.01

4.2. The effect of epicentral distance

We observe that density – related misfits accumulate with distance

Conclusion: We are able to see not only local density effects, but also distant features

It could mean that resolving distant density structures may not be impossible - we could detect density heterogeneities located not only beneath the receiver

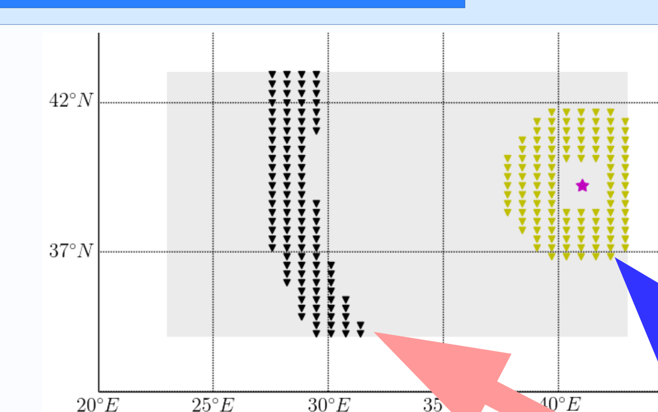
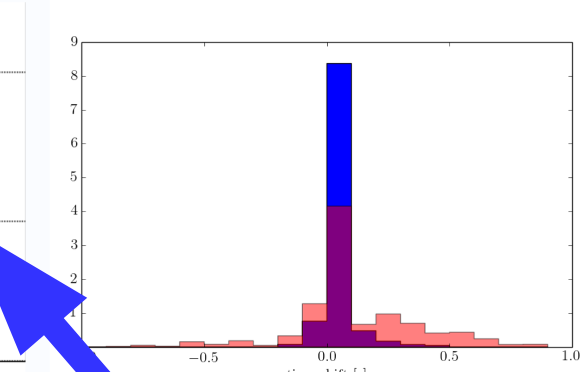
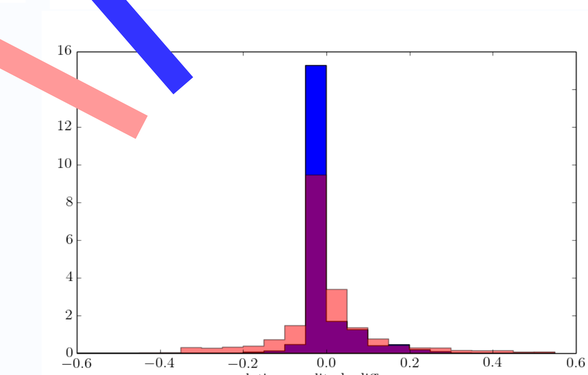


Figure 8 (up): computational grid (grey area) with the source indicated as a magenta star, yellow triangles – stations of distance 100 km – 300 km, black triangles – 1000 km – 1200 km



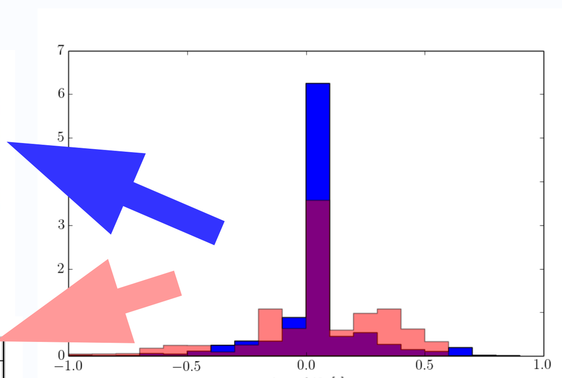


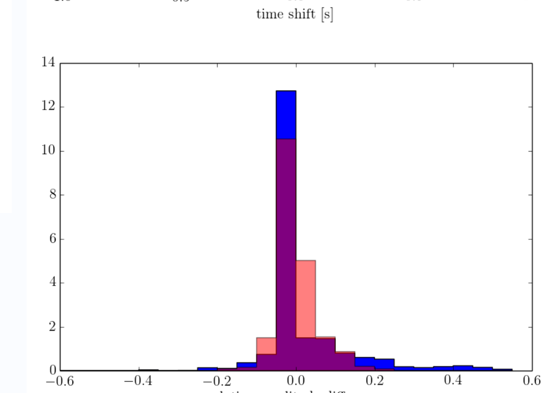
Figures 9, 10 (right): Normed histograms of time shifts (upper figure) and relative amplitude differences (lower figure). Blue: misfits for local stations (yellow triangles), magenta: misfits for distant stations (black triangles). Stacked for 5 experiments with 200 km lateral correlation length

Table 2: Time shift and relative amplitude difference standard deviations for different epicentral distances

epicentral distance	time shift standard deviation	amplitude standard deviation
100 km - 300 km	0.15 s	0.068
1000 km - 1200 km	0.64 s	0.111

4.3. The effect of frequency band





Figures 11, 12 (left): Normed histograms of time shifts (upper figure) and relative amplitude differences (lower figure). Blue: misfits for the highest frequency band (0.02 – 0.125 Hz), magenta: misfits for the lowest frequency band (0.02 – 0.04 Hz). Stacked for 5 reference experiments

For different frequency bands two opposing effects play a role, therefore we do not observe as clear change in the histogram shape as in panels 4.1 and 4.2:

- The more heterogeneities are seen by certain frequency, the more density-related misfits: we observe bigger misfit values for the lower frequency band (especially time shifts?)

- The misfits also grow with the propagation distance (the number of wavelengths traveled between source and receiver) – we observe bigger misfits for the higher frequency band (especially amplitude differences?)

Table 3: Time shift and relative amplitude difference standard deviations for different frequency bands

frequency band	time shift standard deviation	amplitude standard deviation
0.02 Hz - 0.04 Hz	0.38 s	0.07
0.02 Hz - 0.067 Hz	0.20s	0.06
0.02 Hz - 0.125 Hz	0.19 s	0.15

4.4 Does the source-receiver configuration matter?

To answer this question, we use a fixed source-receiver configuration, and look at the misfit functions computed for each of the five different random realizations of the reference medium, for one chosen station.

Random media are uncorrelated with each other, therefore any correlation between the misfits we may observe would be caused by the source-receiver configuration.

We do not see any similarities between the misfit functions (Figure 13), therefore, **the source-receiver configuration does not play a role in the density imprint observation** – the only thing that matters is the medium itself

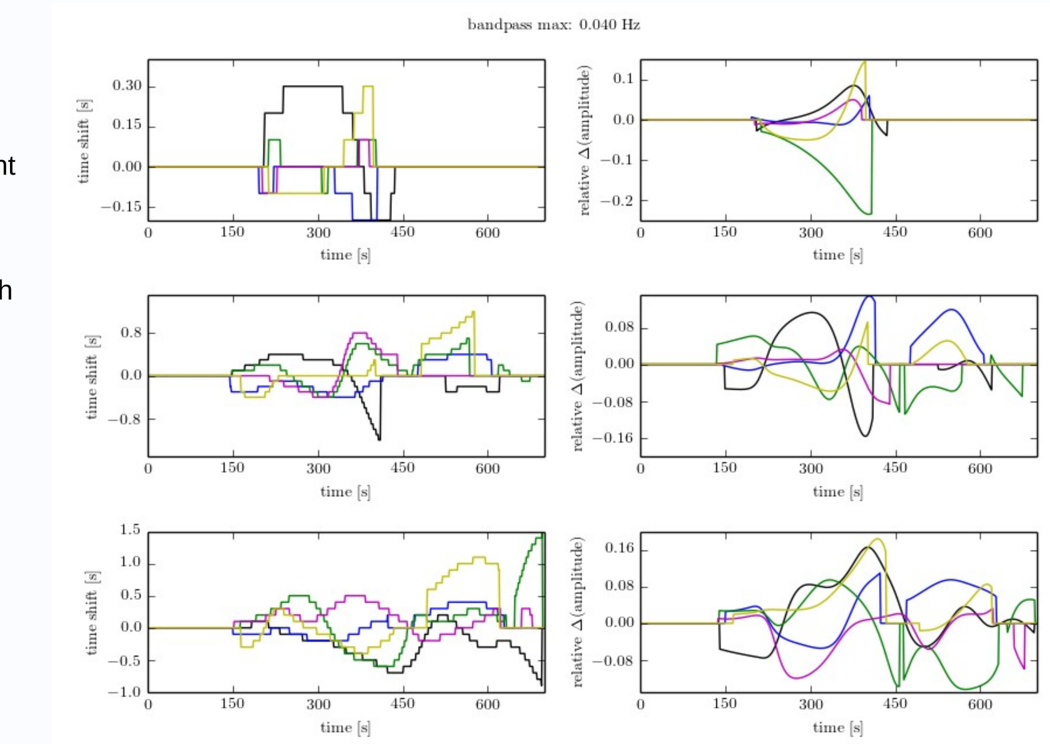


Figure 13: Misfits computed between fully 3D medium and 3D velocity structure. Each color represents a different random realization of a medium. Results for one station, left: time shifts, right: relative amplitude difference. Frequency band 0.02 - 0.04 Hz.

5. Are tomographic models biased if we do not account for density?

Velocity tomography

The density – related bias in velocity tomography is existent, but not significant

Table 4: Velocity bias calculated for certain distance range using time shift standard deviations fromTable 2

	200 km	change of the PREM value for 200 km	1000 km	change of the PREM value for 1000 km
v_p	25.12 $\frac{m}{s}$	0.43%	21.45 $\frac{m}{s}$	0.37%
v_s	8.65 $\frac{m}{s}$	0.27%	7.38 $\frac{m}{s}$	0.23%

Attenuation tomography

If we assume that amplitude changes are attenuation-related (where in fact they are density-related), how would we need to change the q (attenuation) model?

The relation between change in attenuation and change in amplitude (x- epicentral distance, f - frequency and v - velocity (here: shear wave velocity):

$$\delta(q) = -\frac{v}{\pi f x} ln(\delta(ln(A)) - 1)$$

Change in attenuation corresponding to the biggest amplitude misfit: **71% of the model value**. Change in attenuation corresponding to our mean amplitude misfit for 1000-1200 km - **53% of the model value**

Conclusion: attenuation tomography may be massively biased. It could be impossible to distinguish between density and attenuation effects

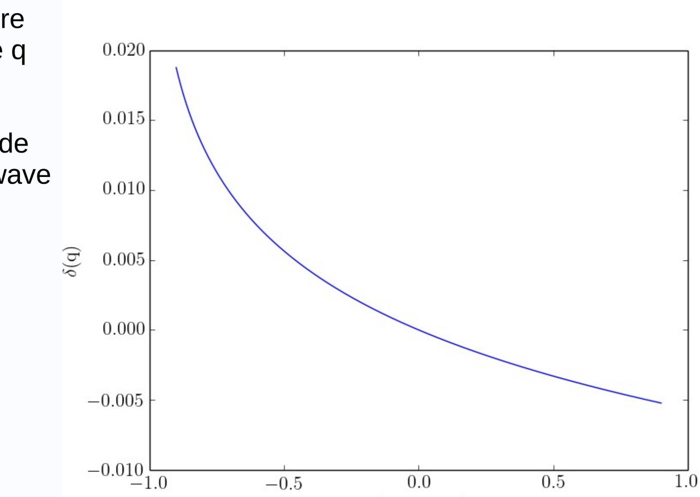


Figure 14: The difference in attenuation that is needed to observe certain change in amplitude. Distance: 1000 km, velocity: 3.2 km/s , frequency: 0.125 Hz

6. Conclusions

- **We do observe significant density imprint of traveltimes and amplitudes on short period seismograms (after the first wave arrival)**

- The more scatterers, the more visible the density imprint

- Density-related misfits accumulate with distance – possibility of resolving distant features

- Velocity tomography is not significantly biased by not accounting for density structure

- Attenuation tomography could be massively biased due to neglected density imprint. It may not be possible to distinguish between attenuation and density

References

1] Trampert J., Fichtner A., 2013. Global imaging of the Earth's deep interior: seismic constraints on (an)isotropy, density and attenuation, in: Physics and chemistry of the deep Earth, edited by Karato S.-I., Wiley-Blackwell, p. 324-350.
[2] Igel H., Gudmundsson O., 1997. Frequency-dependent effects on travel times and waveforms of long – period S and SS waves, in: Physics of the Earth and Planetary Interiors 104, 229-246.
[3] Dziewoński A., Anderson D., 1981. Preliminary reference Earth model, in: Physics of the Earth and Planetary Interiors 25, 297-356.s
[4] Brocher T., 2005. Empirical relations between elastic wavespeeds and density in the Earth's crust, in: Bulletin of the Seismological Society of America, Vol. 95, No. 6, pp. 2081-209

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