The effect of truncating the normal mode coupling equations on synthetic spectra

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Introduction

Seismic waves at very low frequencies (e.g. between 0.1 mHz and 10 mHz) can be used to understand the free oscillations of the Earth. These result from the constructive interference of traveling waves in opposite directions, and can be expressed in terms of the eigenfunctions of the Earth. This infinite set of eigenfunctions constitute a complete mathematical basis, and can be used to calculate synthetic seismograms. In 3D earth models, this is can be done using a procedure known as "*mode coupling*". In order to implement the calculation, it is necessary to select a finite subset of modes (defined as a frequency range) to be considered. This truncation of the infinite-dimensional equations necessarily introduces an error into the results. Here we consider the fundamental question: **if we wish to calculate synthetic spectra in a given frequency range (ω_1, ω_2), how many modes must we couple for the resulting spectra to be sufficiently accurate?**

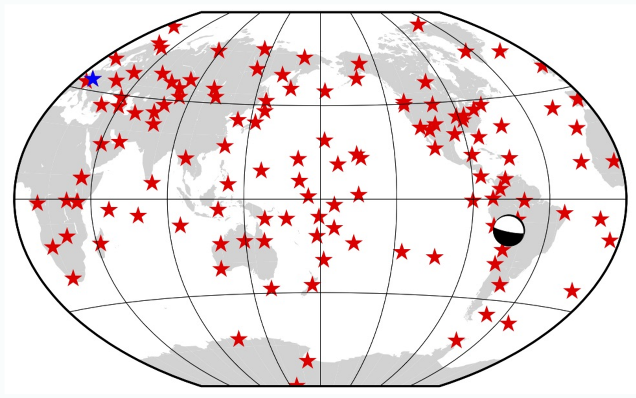
1. Methods

For an SNREI (spherical, non-rotating, elastic, isotropic) earth model, the equations governing free oscillations are reasonably straightforward to solve. Thus, it is possible to compute the normal modes of a 1D model such as PREM. Each mode has a specific frequency, and in the 1D case oscillates independently of all other modes.

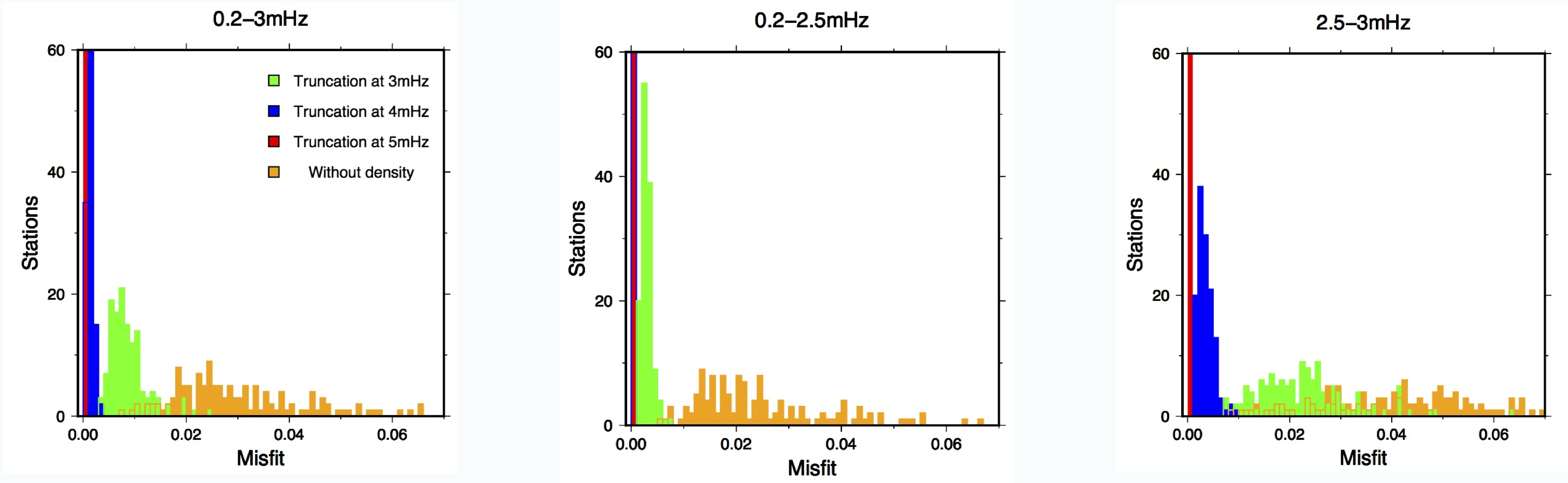
3D effects (including rotation, ellipticity, and lateral heterogeneity) can be taken into account by allowing interaction (energy exchange) between modes. The strongest coupling occurs between modes close in frequency, leading to approximations such as "*self coupling*" and "*group coupling*", but a complete treatment requires us to allow each mode to interact with every other mode. However, numerical implementation of this requires us to work with a finite set of spherical earth eigenfunctions, neglecting coupling with modes outside that set.

To investigate this issue, we compute spectra in 3D models up to 3 mHz with ellipticity and rotation, but allowing coupling with all modes up to 6 mHz. We treat these as "*reference*" spectra. We then investigate how the spectra change as we reduce the upper frequency used in coupling. We compare this to the effect of removing lateral variations in density from the model.

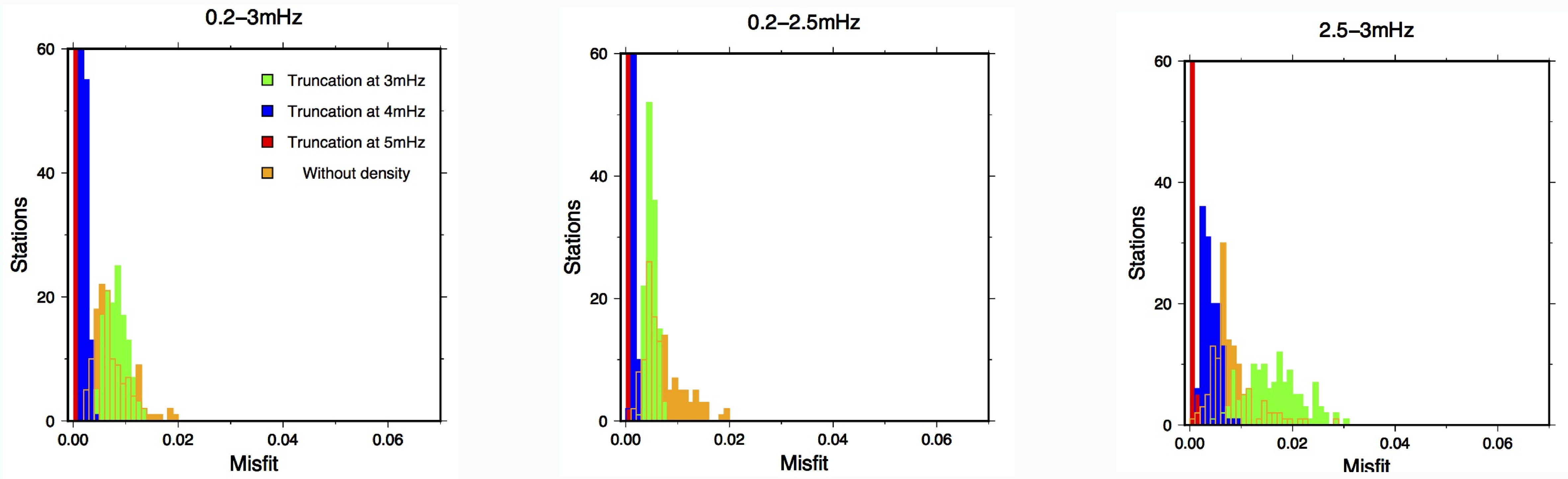
3. Histograms of truncation errors



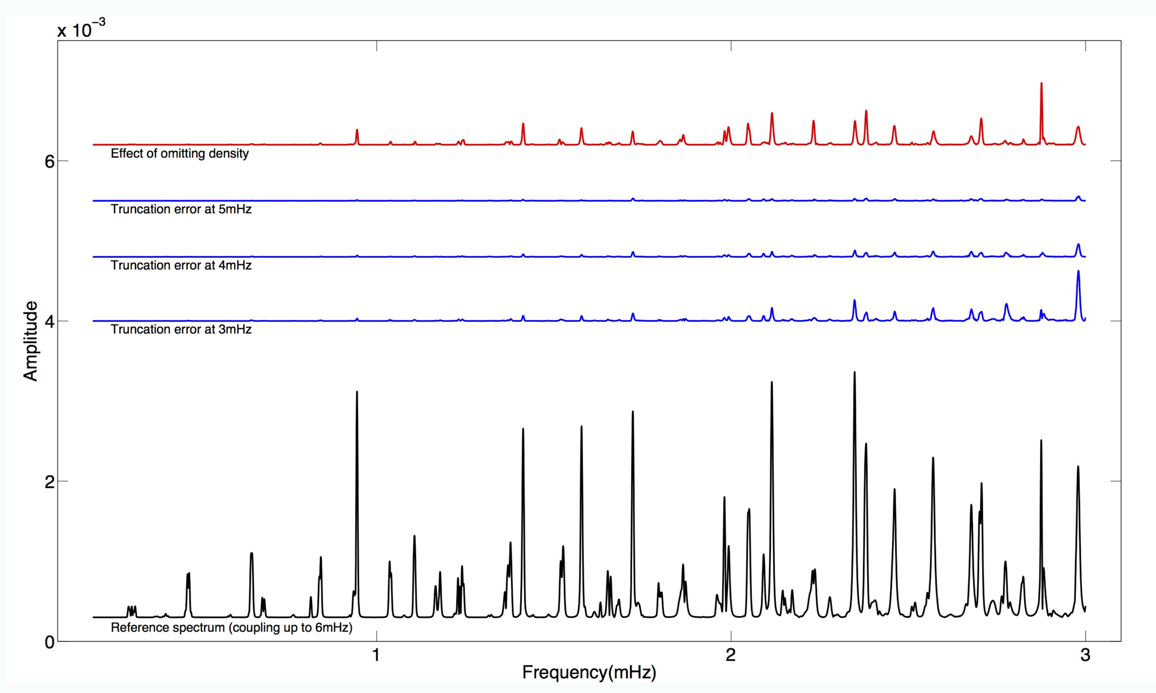
Below: Histograms of misfit between reference spectra and others (see legend) at a global distribution of stations (see map). We show one histogram for the range 0.2–3 mHz, and also the same dataset in the ranges 0.2–2.5 mHz and 2.5–3 mHz. It is apparent that coupling at least up to ~4 mHz is necessary, depending on the input model, if we wish to image Earth's density structure using observations up to 3 mHz.



We repeat the calculations but instead of S20RTS we use a model composed of random numbers. While S20RTS has well known correlation lengths horizontally and vertically, the random numbers we generated have not. As in S20RTS V_p and ρ are scaled versions of the V_s structure. This results in a model with significantly stronger short-wavelength structure compared to S20RTS, but without horizontal or vertical correlation. We see that the latter reduces the effect of density since that is some integral of the model with the eigenfunctions over space. Because the signal is smaller, the relative effect of the truncation errors becomes more pronounced.



2. Example of spectra



Synthetic spectra are calculated using full mode coupling for the 1994 Bolivia earthquake. We use the Iterative Direct Solution Method (Al-Attar *et al.*, 2012) to compute individual spectra for the 3D mantle model S20RTS (Ritsema *et al.*, 1999).

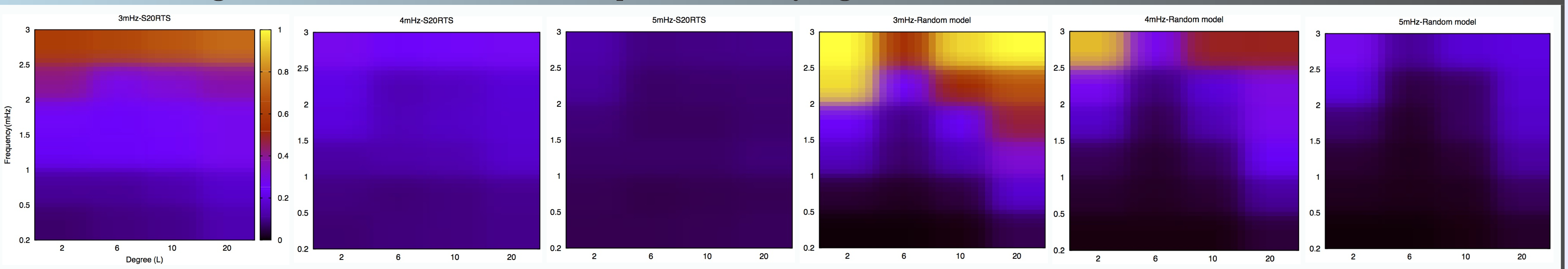
The truncation error is the misfit between the reference spectra and those truncated at lower cut-off frequencies. The density effect is the misfit between 6mHz spectra with and without density. The misfit is defined as

$$\chi(\omega_c) = \sum_\omega \sqrt{\left(S(\omega_c,\omega) - S_{ref}(\omega)\right)^* \left(S(\omega_c,\omega) - S_{ref}(\omega)\right)}.$$

Truncation errors can be significant compared to the contribution from density. Thus accurate imaging of Earth's density structure requires coupling to well above the frequency range of interest.

Black: Reference spectra (complete up to 6 mHz);
Blue: Truncation error introduced by coupling to lower frequencies than 6mHz;
Red: Effect of removing density variations from S20RTS (i.e., incorporating V_s and V_p heterogeneity only; in S20RTS, V_p and ρ are scaled to V_s);

4. Relative changes in truncation error with respect to density signal



Above: The relative misfit is computed by dividing the misfit from truncation by the misfit from neglecting density. This is done in a certain frequency range and for a model up to a certain spherical harmonic degree. This relative misfit is calculated for each seismic station and then averaged over all stations. It is clear that the truncation error increases for increasing frequency and that the truncation level depends on the model power and correlation lengths.

Conclusion

- Truncating the normal mode coupling equations introduces error into synthetic spectra;
- For accurate imaging of earth structure, these truncation errors must be negligible compared to effects due to heterogeneity;
- Truncation errors grow as we approach the cutoff frequency;
- It is not sufficient to simply couple modes in the frequency range of interest;
- For accurate spectra at 3 mHz, it appears that coupling to at least ~4mHz may be necessary (for S20RTS-like structure), but the exact number depends on the correlation lengths of the model;

References

Al-Attar, D. Woodhouse, J. H. Deuss, A., 2012. Calculation of normal mode spectra in laterally heterogeneous earth models using an iterative direct solution method, *Geophys. J. Int.*, 189, 1038-1046.

Ritsema, J., van Heijst, H.-J., Woodhouse, J.H., 1999. Complex shear wave velocity structure imaged beneath Africa and Iceland, *Science*, 286, 1925–1928.