The influence of elasticity in the lithosphere on subduction evolution, a numerical study

Loes Cornelis[1], Cedric Thieulot[1]

[1]Department of Earth Sciences, Utrecht University

Universiteit Utrecht

1. Introduction

A common approach towards describing the rheology of the Earth when modelling plate tectonics is to characterize the mantle and lithosphere by a viscoplastic rheology. In order to accurately describe the Earth's rheology, one has to realise that both exhibit **elastic properties**.
The mantle behaves elastically on a short time scale, as with seismic waves, and viscously on a geologic timescale. The lithosphere acts as an elastic shell which can store stresses over geological timescales. Several studies have shown that elasticity is likely to play an important role during subduction [1][2][3] .

To determine the influence of elasticity on subduction evolution, a new numerical finite element thermo-mechanically coupled visco-elastic code based on the marker-and-cell technique has been developed.

2. A formulation for viscoelasticity

For the implementation of a viscoelastic rheology, a **Maxwell viscoelastic model** is used. This model assumes a viscoelastic material to react as an elastic and viscous element connected in series (shown in Fig. 1). This results in the strain rate tensor being a summation of the elastic strain rate tensor $\dot{\boldsymbol{\varepsilon}}^e$ and the viscous strain rate tensor $\dot{\boldsymbol{\varepsilon}}^v$ (1).

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^v \quad (1)$$

$$\boldsymbol{\nabla} \cdot \{-p\boldsymbol{I} + \boldsymbol{\tau}\} = \rho \boldsymbol{g} \quad (2)$$

$$p = -\lambda \boldsymbol{\nabla} \cdot \boldsymbol{v} \quad (3)$$

$$\boldsymbol{\tau}^{t+\Delta t} = 2\eta_{eff}\dot{\boldsymbol{\varepsilon}}^{\,t+\Delta t} + \frac{\eta_{eff}}{\mu \Delta t}\boldsymbol{\tau}^t + \frac{\eta_{eff}}{\mu}\left(\boldsymbol{\omega}^t\boldsymbol{\tau}^t - \boldsymbol{\tau}^t\boldsymbol{\omega}^t\right) \quad (4)$$

$$\{\mathbf{K}(\lambda) + \mathbf{K}(\eta_{eff})\}\,\mathbf{V}^{t+\Delta t} = \mathbf{F} - \mathbf{R}^t \quad (5)$$

(2) momentum conservation equation
(3) penalty formulation relating pressure to strain rate
(4) viscoelastic expression for the deviatoric stress
(5) finite element expression used to solve for the velocity field $\mathbf{V}$

Here, p is the pressure, λ the penalty factor, $\boldsymbol{v}$ the velocity, τ the deviatoric stress, ω the rotation rate and

$$\eta_{eff} = \eta \frac{\Delta t}{\Delta t + \frac{\eta}{\mu}} \quad (6)$$

is the **effective elastic viscosity**. η is the prescribed fluid viscosity and μ the shear modulus.
In the finite element expression for viscoelasticity (5), $\mathbf{K}$ are finite element matrices dependent on either λ or η_{eff}, and $\mathbf{F}$ and $\mathbf{R}^t$ are the finite element matrices representing forcings due to gravity and elastically stored stresses.

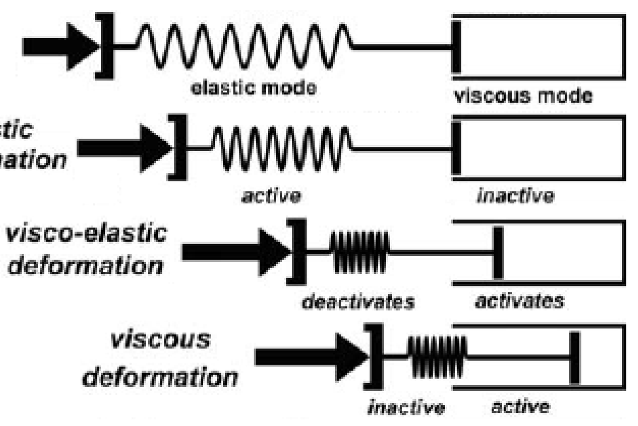

Fig. 1: The response of a Maxwell viscoelastic model to an applied stress. Top to bottom: The material initially behaves elastic, shown by the shortening of the spring. The viscous component then activates and becomes the dominant response. Adapted from [4].

5. Conclusions and Outlook

Several benchmarks have successfully been performed and the viscoelastic implementation can be used to test the influence of elasticity on **subduction evolution**. A set-up by [5] will be used to study the influence of elasticity on **unbending of the subducting slab** in the mantle (Fig. 2).

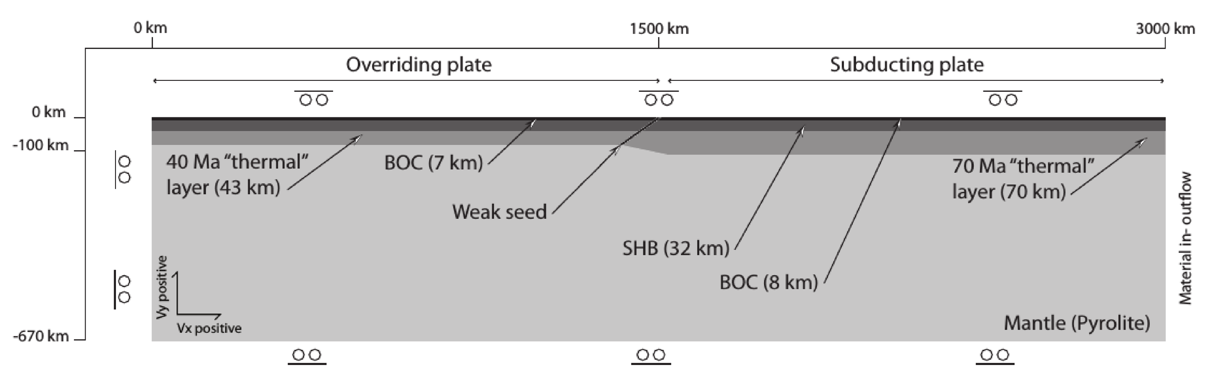

Fig. 2: **A subduction set-up from [5].**

[1] Farrington, R., et al. *The role of viscoelasticity in subducting plates.* G^3, (2014)
[2] Fourel, L., Goes, S., and Morra, G. *The role of elasticity in slab bending.* G^3, (2014)
[3] Funiciello, F., et al. *Dynamics of retreating slabs 1* JGR: Solid Earth, (2003)
[4] Gerya, T. *Introduction to numerical geodynamic modelling.* Cambridge University Press, (2009)
[5] Quinquis M.E.T., et al. *A numerical comparison study of thermal−mechanical subduction.* Manuscript in preparation

3. The effective elastic viscosity η_{eff}

The occurrence of the effective elastic viscosity instead of the fluid viscosity in the finite element equation has a significant influence on the model results.

The finite element equation (5) for the **viscous case** has $\mathbf{R}^t = 0$ and uses the **fluid viscosity** $\boldsymbol{K}(\eta)$ instead of the **effective elastic viscosity** $\boldsymbol{K}(\eta_{eff})$. Since the effective elastic viscosity depends on the used **time step** and **shear modulus**, it differs from the fluid viscosity. Thus its use has a significant effect on material behaviour. (Fig. 3).
When looking at the inset in Fig. 3 the influence of the time step on a prescribed lithosphere/mantle system when using the effective elastic viscosity becomes clear.

- *Upper case:* It is evident that the lithosphere and mantle now have **similar effective elastic viscosities** and will therefore behave alike. This is in stark contrast to the **three orders of magnitude** difference which was initially prescribed.
- *Bottom case:* the viscosity difference is larger, but still **an order of magnitude smaller** than initially prescribed.

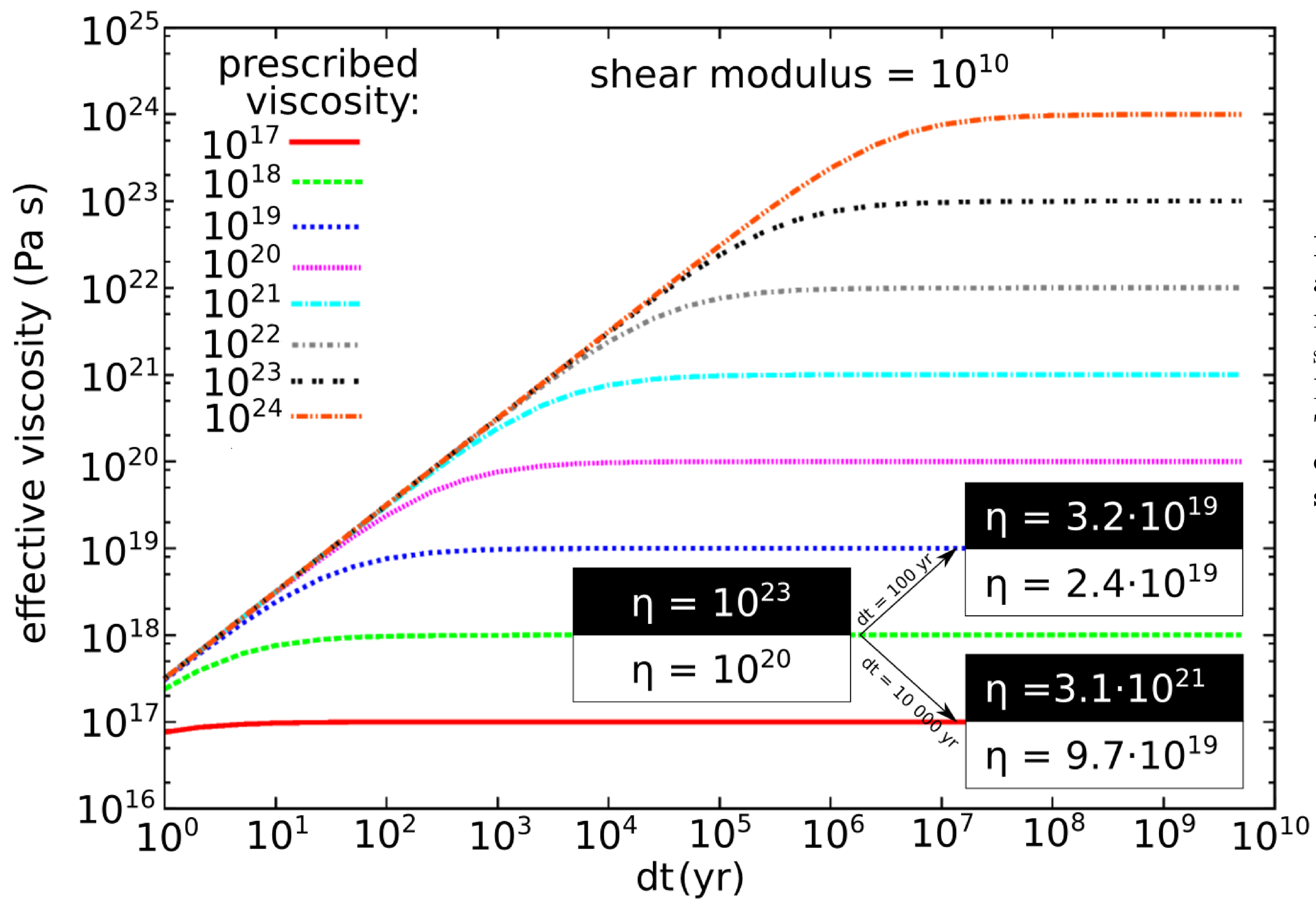

Fig 3: The effect of the time step (x-axis) on the effective viscosity. A shear modulus of $10^{10} Pa$ has been used. Different prescribed fluid viscosities are plotted by different lines.
The inset shows the effect of the choice of the time step for a specific lithosphere/mantle case:
- The viscosities on the left are the prescribed fluid viscosities (used in a viscous model).
- The viscosities on the right are the effective elastic viscosities which are used to solve for the velocity field in a viscoelastic model.

The effect of the effective elastic viscosity is also visible in Fig. 4. The prescribed fluid viscosities indicate a sturdy beam in a weak medium. When looking at the effective elastic viscosity, the beam is now weaker than the medium. This will alter the outcome significantly compared to the case using the fluid viscosities.

4. Benchmarking the elasticity implementation

A benchmark done by [4], the sinking beam benchmark, is performed to test the implementation of elasticity.

- An unstressed beam is placed in a medium and attached to the left wall.
- A gravity of $g_y = 10m/s$ is applied to the domain causing the beam to bend.
- The gravity is set to zero after 20 Kyrs, resulting in a return of the beam to its original position due to its elastic properties

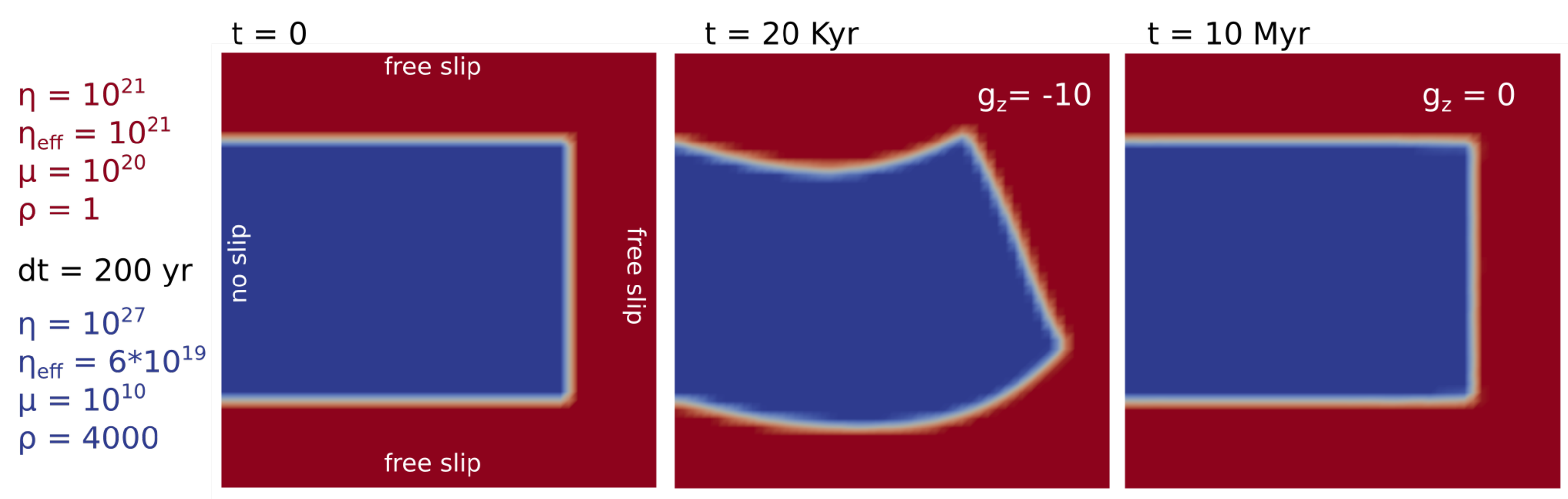

Fig. 4: The results of the sinking beam benchmark. From left to right:
1. The original configuration of the slab.
2. The shape of the slab after 20 Kyr.
3. The return of the slab to its initial position after setting the gravity field to zero.

The evolution of the sinking beam through time is shown in Fig. 4. The beam initially bends under the influence of gravity after which it returns to its original position when gravity is set to zero.
The maximum velocity in the domain is given in Fig. 5. The results are in agreement with those obtained by [4], therefore this benchmark has been performed successfully.

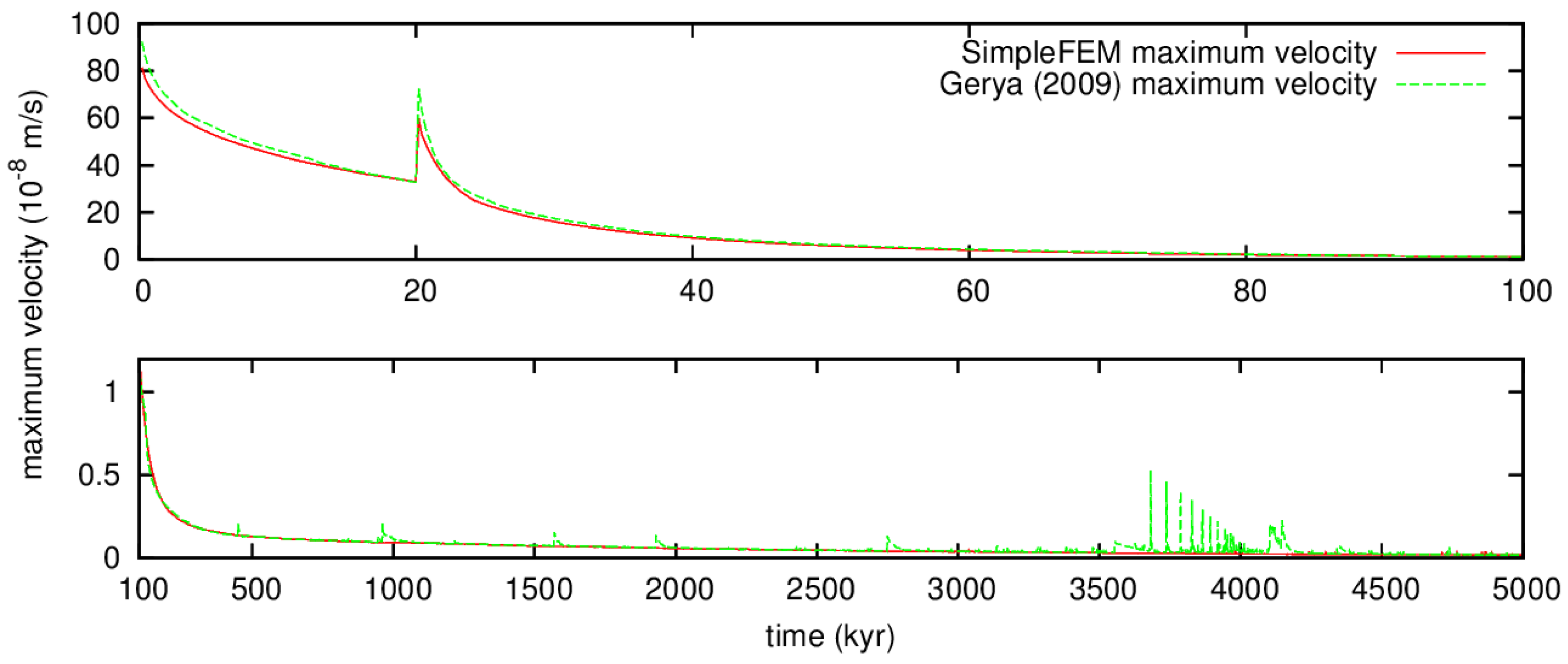

Fig. 5: The maximum velocity as a function of time. After 20 Kyr, the gravity is set to zero and the beam returns to its original position. It can be seen that the maximum velocity slowly converges to zero. The results of the code developed are smoother than the results from [4].

LaTeX TikZposter