

# Analytic clear-sky index fluctuation correlation model

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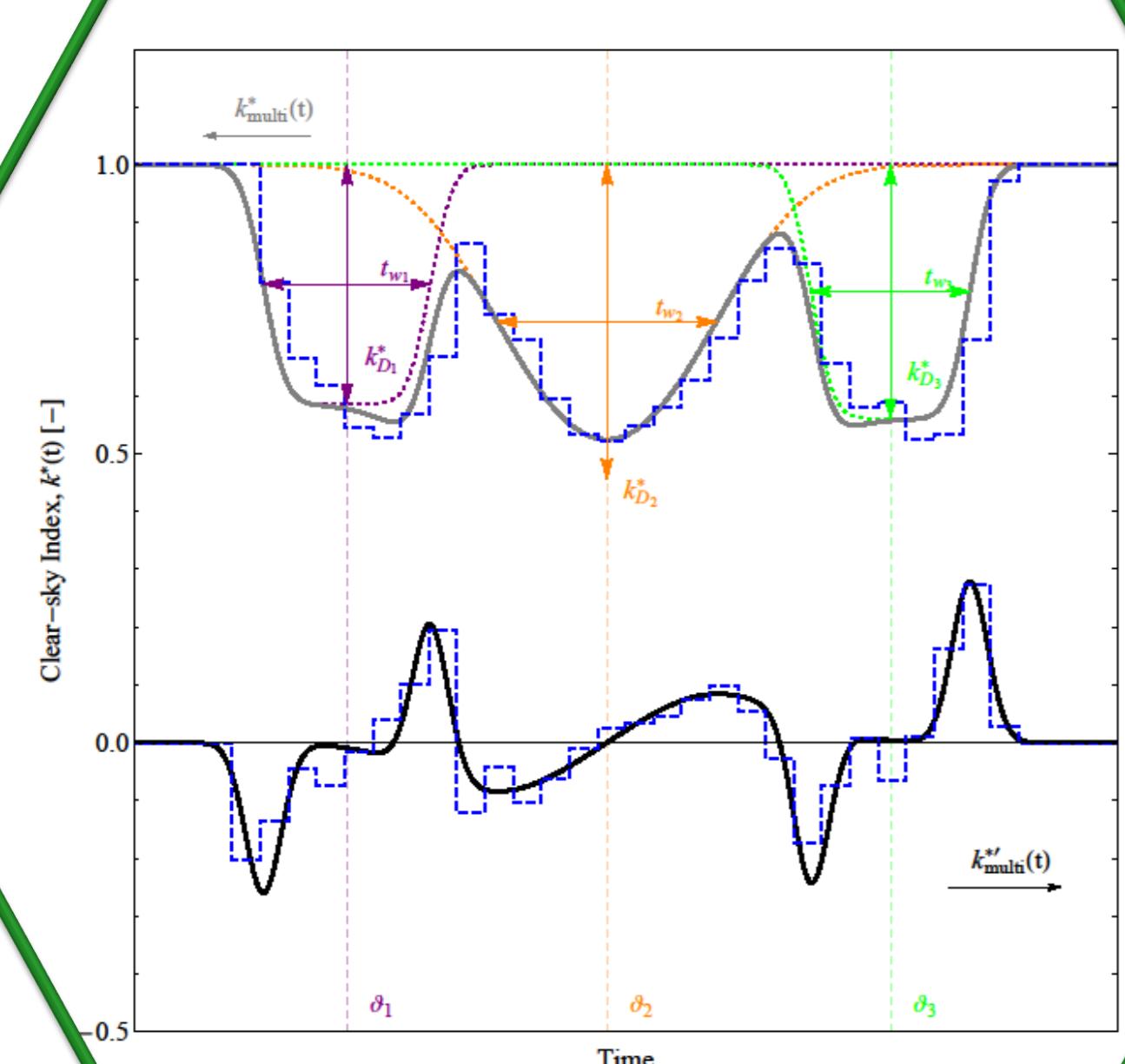
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## Set-up

Bottom-up calculation of lagged **cross-correlation** versus time between PV-systems due to multiple quasi-statically moving clouds, based on [1].

**Sigmoid-shaped** parametrization of clear-sky index  $k_t^*$  time series of clouds, with **Gaussian** first derivative.

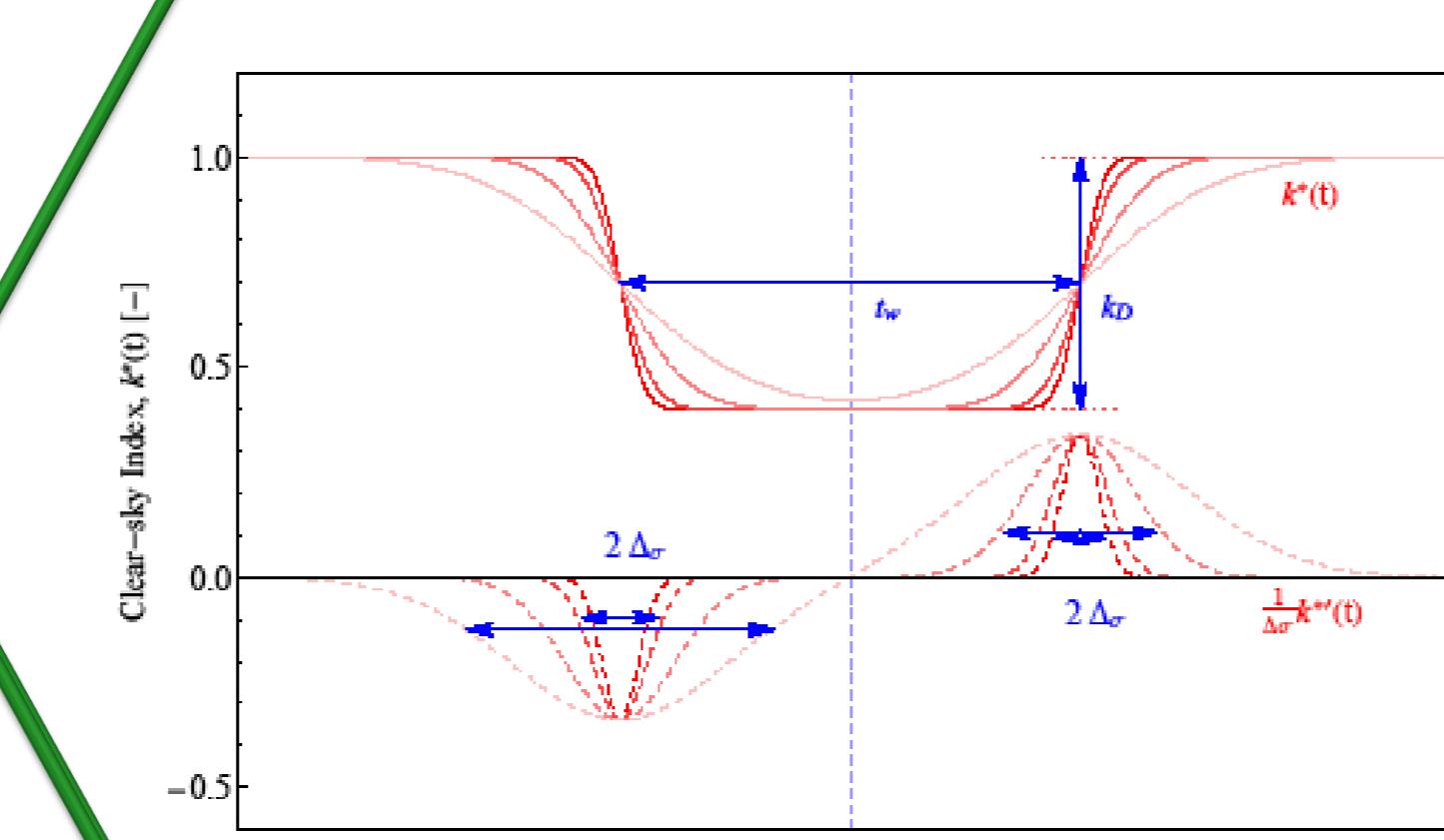
Direct relation of (auto-) correlation function and shape parameters



**$N_c$ -cloud (multi): Linear combination of Cross-correlations**

$$\rho_{N_c}(\tau) = \frac{(k_{\text{multi}}^{*\prime} \star k_{\text{multi}}^{*\prime})(\tau)}{(k_{\text{multi}}^{*\prime} \star k_{\text{multi}}^{*\prime})(0)} = \frac{\sum_{m=1,n=1}^{N_c} (k_m^{*\prime} \star k_n^{*\prime})(\tau)}{\sum_{m=1,n=1}^{N_c} (k_m^{*\prime} \star k_n^{*\prime})(0)}$$

**$N_c - 1$  unique parameters for width ( $t_w$ ), lag ( $\theta$ ), opacity ( $k_D$ )**



**1-cloud Sigmoid shape clouds, Gaussian derivatives**

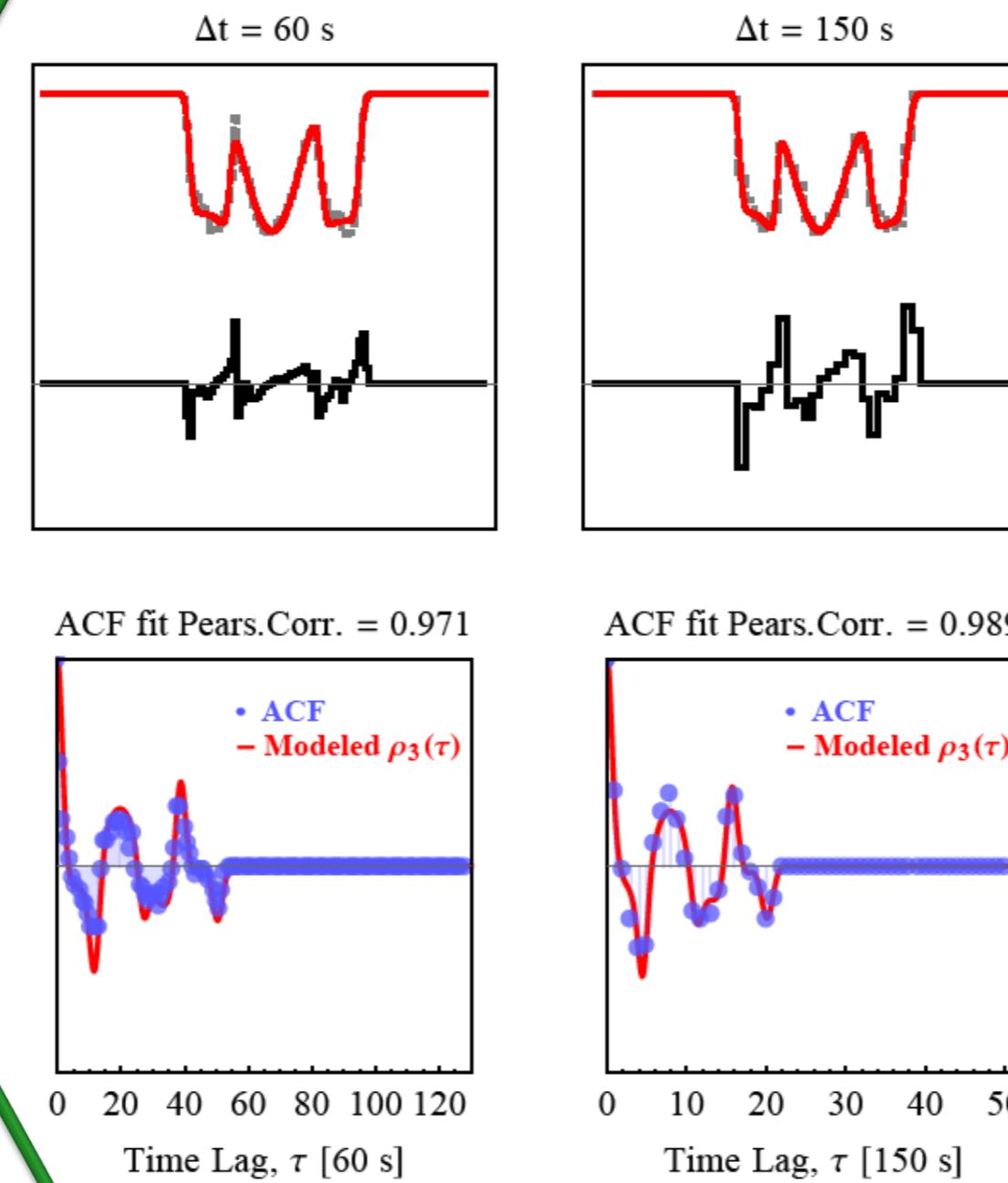
$$k^*(t) = k_D^* \left[ \frac{1}{2} \left[ \text{Erf} \left( \frac{t - t_w}{\Delta\sigma} \right) + \text{Erfc} \left( \frac{t + t_w}{\Delta\sigma} \right) + 1 \right] - 1 \right] + 1$$

$$k^{*\prime}(t) = \frac{k_D^*}{\sqrt{\pi}\Delta\sigma} \left[ \text{Exp} \left( -\left[ \frac{t - t_w}{\Delta\sigma} \right]^2 \right) - \text{Exp} \left( -\left[ \frac{t + t_w}{\Delta\sigma} \right]^2 \right) \right]$$

**$N$ -cloud (multi): lagged summation**

$$k^{\text{multi}}(t) = 1 + \sum_{m=1}^{N_c} [k^*(t - \theta_m) - 1]$$

**Numerical Comparison**



## References

[1] B. Elsinga and W.G.J.H.M. van Sark, Sol. Energy (155) 2017, p. 985-1001

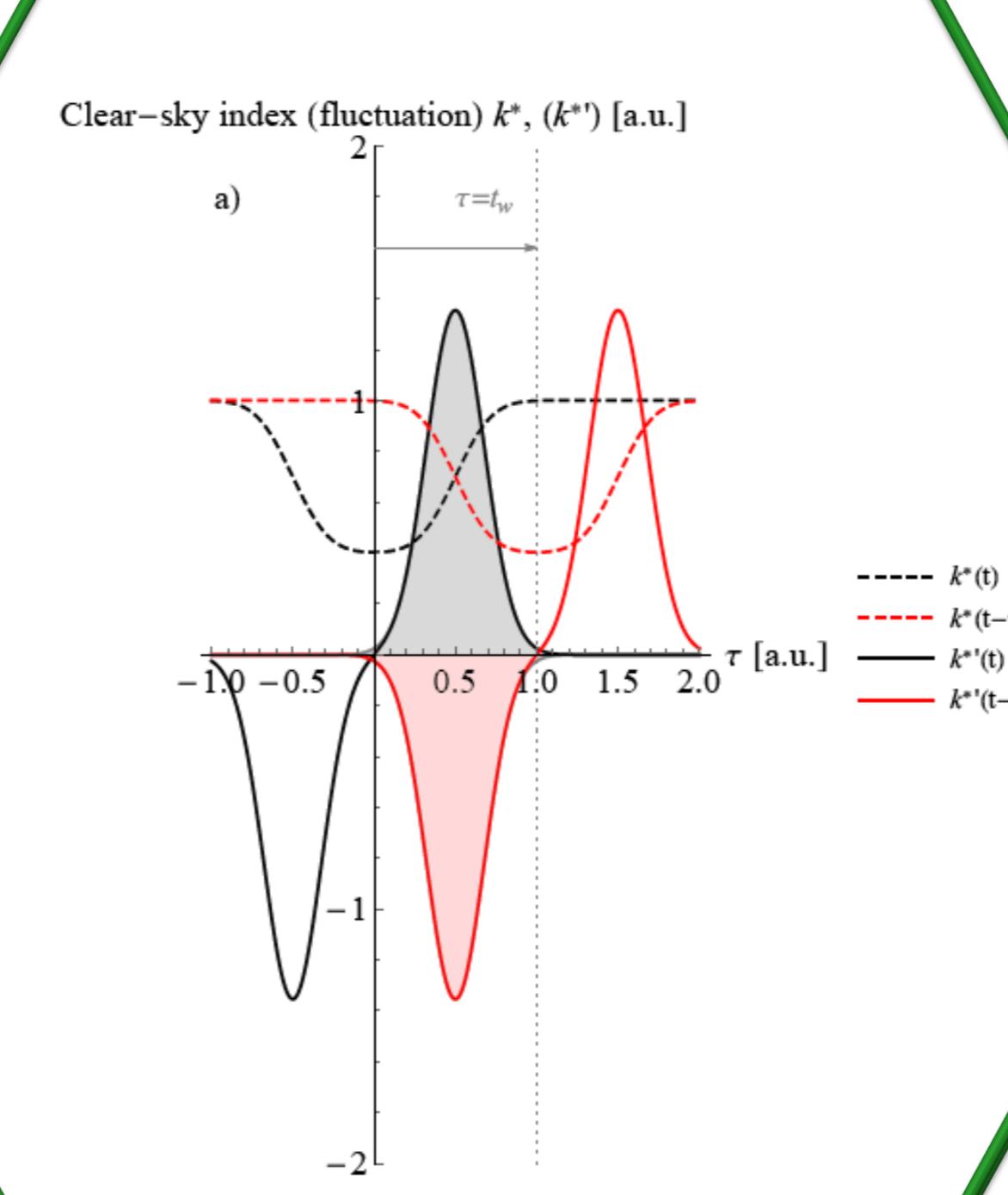
[2] J. Widén, Sol. Energy (122) 2015, p.1409-1424

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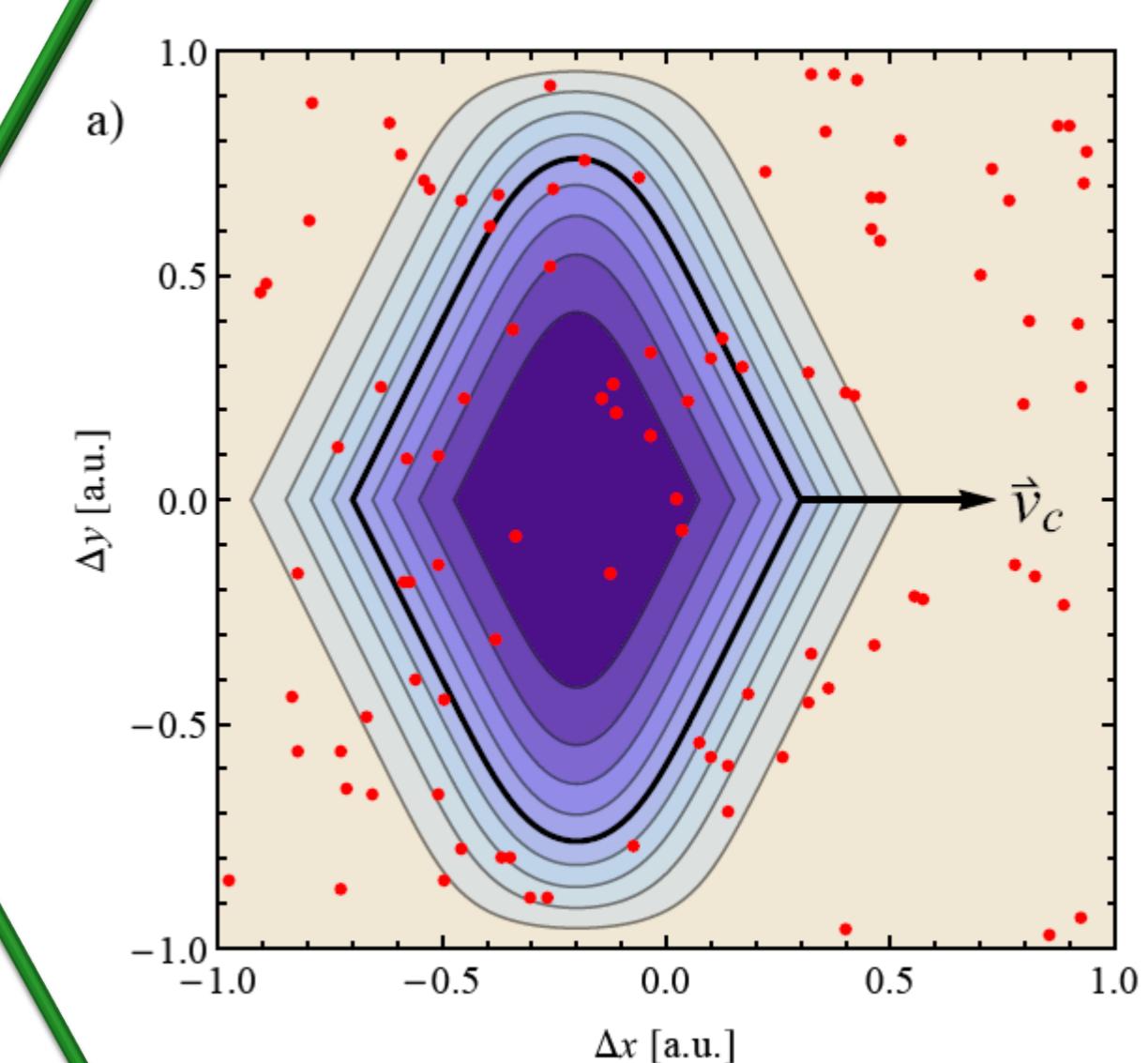


$$\rho(\tau) = \frac{\int_{-\infty}^{\infty} (k^{*\prime}(t)k^{*\prime}(t-\tau)) dt}{\sqrt{\int_{-\infty}^{\infty} k^{*\prime}(t)^2 dt} \sqrt{\int_{-\infty}^{\infty} k^{*\prime}(t)^2 dt}}$$

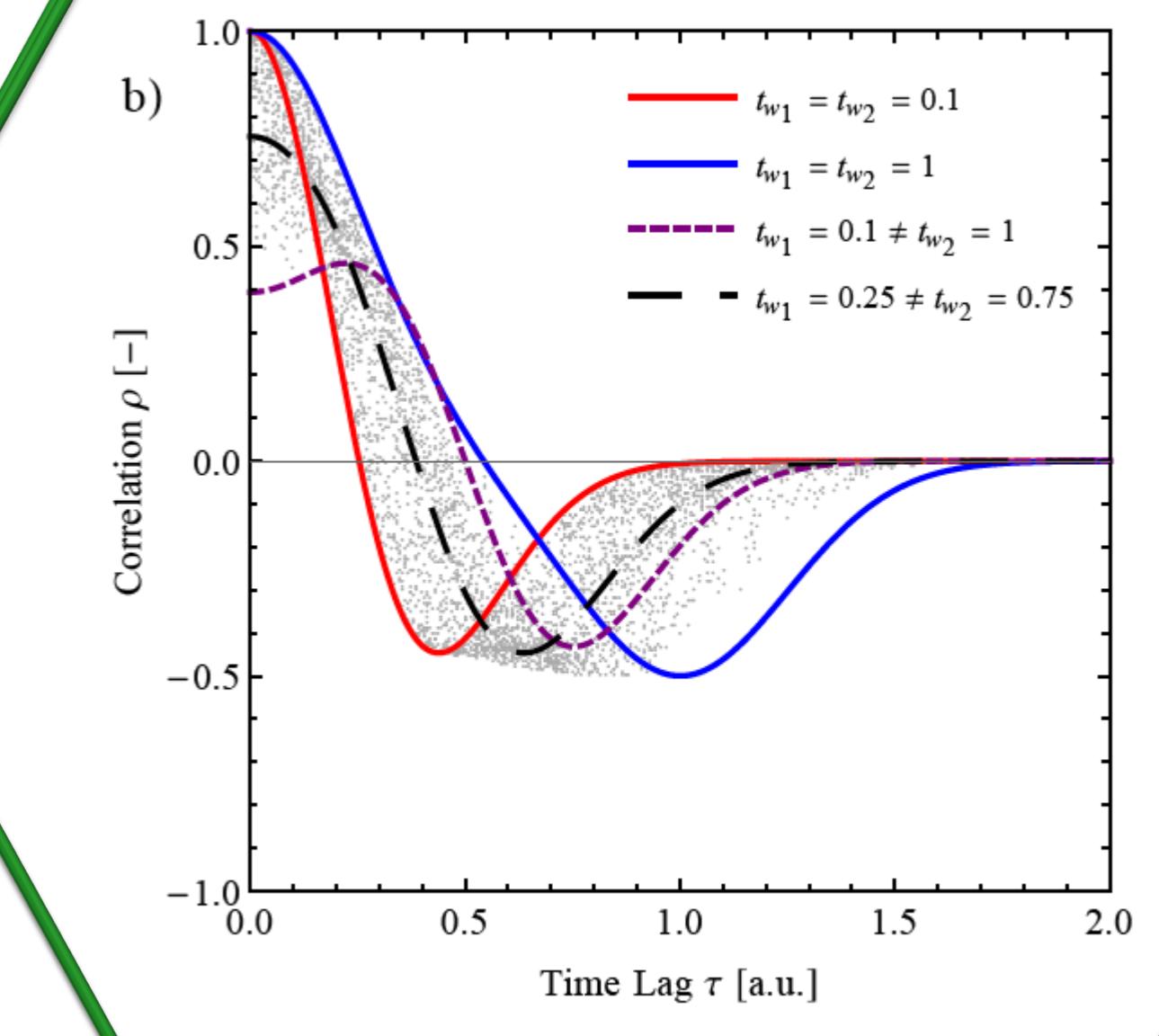
$$= \frac{\int_{-\infty}^{\infty} k^{*\prime}(t)k^{*\prime}(t-\tau) dt}{\int_{-\infty}^{\infty} k^{*\prime}(t)k^{*\prime}(t-0) dt} \equiv \frac{(k^{*\prime} \star k^{*\prime})(\tau)}{(k^{*\prime} \star k^{*\prime})(0)}$$

$$\rho(\tau) = \frac{\text{Exp} \left( -\frac{1}{2} \left[ \frac{\tau - t_w}{\Delta\sigma} \right]^2 \right) - 2 \text{Exp} \left( -\frac{1}{2} \left[ \frac{\tau}{\Delta\sigma} \right]^2 \right) + \text{Exp} \left( -\frac{1}{2} \left[ \frac{\tau + t_w}{\Delta\sigma} \right]^2 \right)}{2 \left[ \text{Exp} \left( -\frac{1}{2} \left[ \frac{t_w}{\Delta\sigma} \right]^2 \right) - 1 \right]}$$

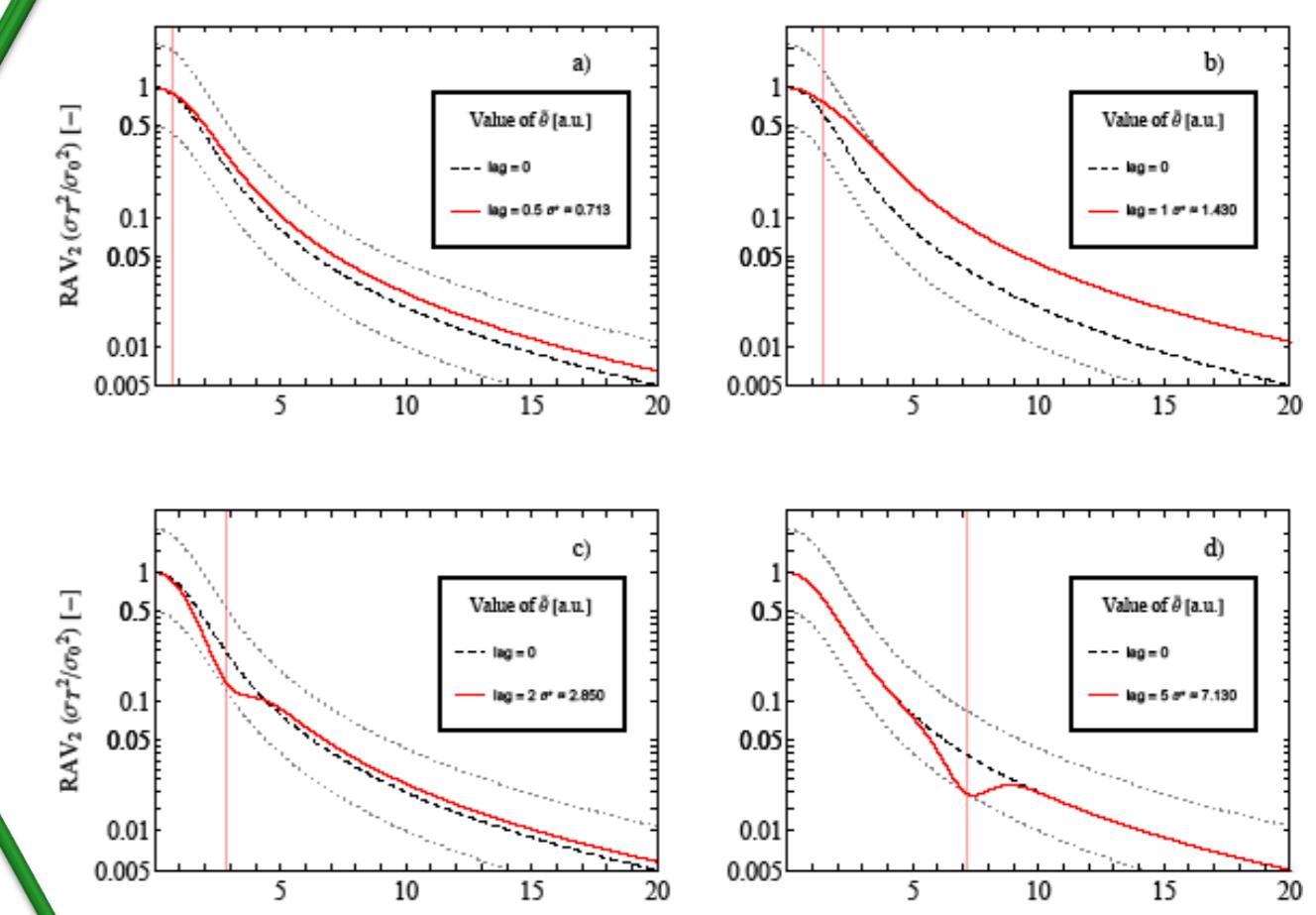
**1-cloud cross-correlation = lagged autocorrelation (\*) evaluated via Fourier-transform**



**2D-cloud and Random Virtual Network: transections**



**Relative Aggregate Variability RAV: see [1,2]**



$$\frac{\sigma_T^2}{\sigma_0^2} = \frac{2}{T^2} \int_0^T [T - \tau] \rho(\tau) d\tau$$

## Key Findings

**Generic** analytic autocorrelation model for parametrized clouds,

Realistic implementation is **trade-off** between detail and number of parameters.

**Flexible** implementation for realistic (dynamic) clouds

Parametrization of aggregate **variability** in PV-network