3D thermal convection in a spherical shell with a free surface

Ludovic Jeanniot[1] and Cedric Thieulot[1]

[1]Earth Sciences - Mantle Dynamics & Theoretical Geophysics,
Utrecht University, The Netherlands
l.jeanniot@uu.nl

1 INTRODUCTION:

Simple symmetric modes of convection of the Earth's mantle can be used to validate numerical codes and predict global dynamics patterns. We use an incompressible and isoviscous Rayleigh-Bénard thermal convection benchmark at infinite Prandtl number in a spherical shell to compare the state-of-the-art finite element code ASPECT with the previously benchmarked numerical code CitcomS.

We explore the sensitivity of the initial and steady-state root mean square velocity, average temperature and Nusselt number measured at both inner and outer boundaries to resolution for a range of Rayleigh numbers. Although the original benchmark is formulated with free slip boundary conditions, we explore for the first time the effect of a free surface on such calculations and compare the measured topography to dynamic topography measurements.

What this poster shows:

3. sensitivity test (free slip)	ASPECT vs CitcomS
4. model resolution test (free surface)	ASPECT
5. free slip vs free surface:	critical Rayleigh
6. perspectives – work in progress:	S40RTS – free surface

2 MODEL SETUP:

The governing equations describe an incompressible Boussinesq fluid at infinite Prandtl number in a 3-D spherical shell that is heated from below and cooled from above:

$$\nabla \cdot \boldsymbol{v} = 0 \tag{1}$$

$$-\nabla p + \nabla \cdot (2\mu\dot{\varepsilon}) = \rho(T)\boldsymbol{g} \tag{2}$$

$$\rho_0 c_p \left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) \tag{3}$$

with $\boldsymbol{v}$ the velocity, p the dynamic pressure, μ the viscosity, $\dot{\varepsilon}$ the strain-rate tensor, ρ_0 the reference mass density, $\boldsymbol{g}$ the gravitational acceleration, C_p the specific heat, T the absolute temperature, t the time and k the thermal conductivity.

The density in Eq. (2) depends on temperature as follows:

$$\rho(T) = \rho_0(1 - \alpha(T - T_0)) \tag{4}$$

where α is the thermal expansion coefficient and T_0 a reference temperature which is set to zero in this work.

Free slip on both the inner and the outer boundary are applied. Temperature boundary conditions are $T(r = R_b, \theta, \phi) = 1$ and $T(r = R_t, \theta, \phi) = 0$ with $R_b = 11/9 \simeq 1.222$ and $R_t = 20/9 \simeq 2.222$ so that the mantle depth is $\Delta R = R_t - T_t = 1$.

The dynamics of the fluid are governed entirely by the Rayleigh number Ra, which can be interpreted as a ratio of the destabilizing force due to the buoyancy of the heated fluid to the stabilizing force due to the viscosity of the fluid and heat transfer by conduction. It is defined as:

$$Ra = \frac{\rho_0 \alpha g \Delta T (\Delta R)^3}{\kappa \mu} \tag{5}$$

where $\kappa = k/\rho_0 c_p$.

The initial temperature is given by the sum of a conductive profile and a perturbation Arrial et al. (2014):

$$T(r, \theta, \phi) = T_C(r) + T_P(\theta, \phi) \sin\left(\pi \frac{r - R_t}{R_t - R_i} \right) \tag{6}$$

with

$$T_C(r) = \frac{R_b(r - R_t)}{r(R_b - R_t)} \tag{7}$$

$$T_P(r, \theta, \phi) = Y_l^{m=0}(\theta, \phi) + \epsilon Y_l^{m \neq 0}(\theta, \phi) \tag{8}$$

The function Y_l^m denotes the normalised spherical harmonic of degree l and order m. Note that $m = 0$ indicates an axisymmetric function. In this work, $\theta \in [0 : pi]$ is the colatitude while $\phi \in [-pi : pi]$ is the longitude.

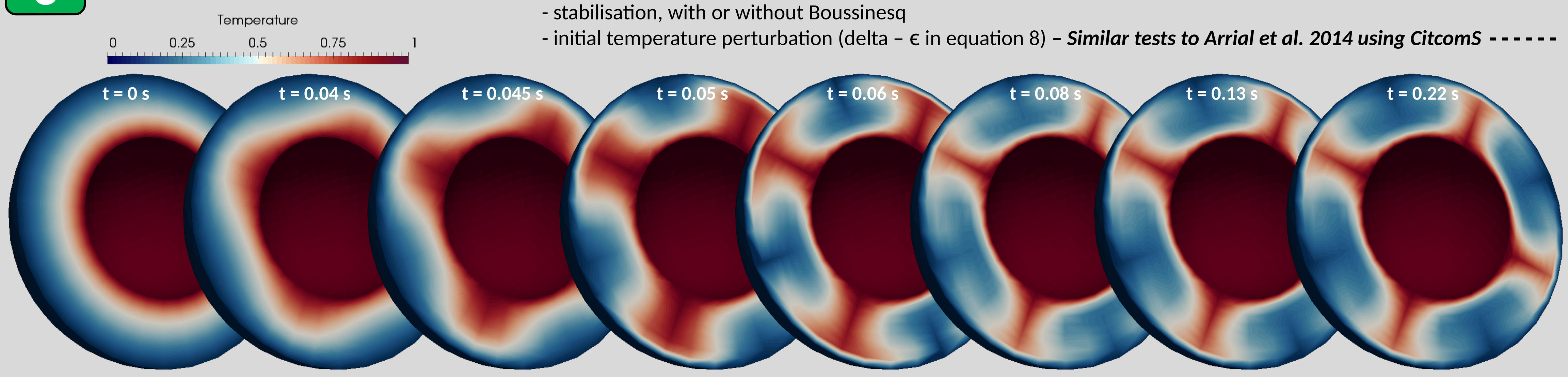
We have

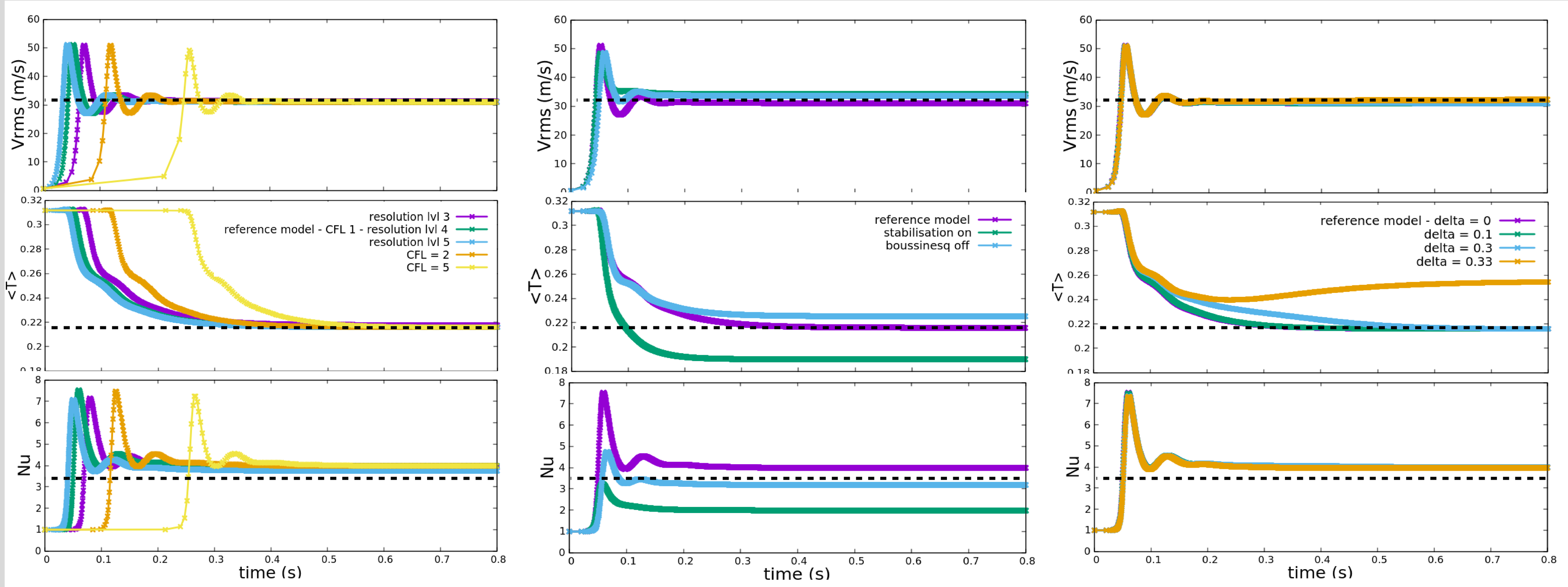
$$Y_l^m(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{2\pi(1+\delta_{m0})(l+m)!}} P_l^m(\cos\theta) \cos(m\phi) \tag{9}$$

where P_l^m are the (unnormalized) associated Legendre functions and δ_{m0} is the Kronecker delta.

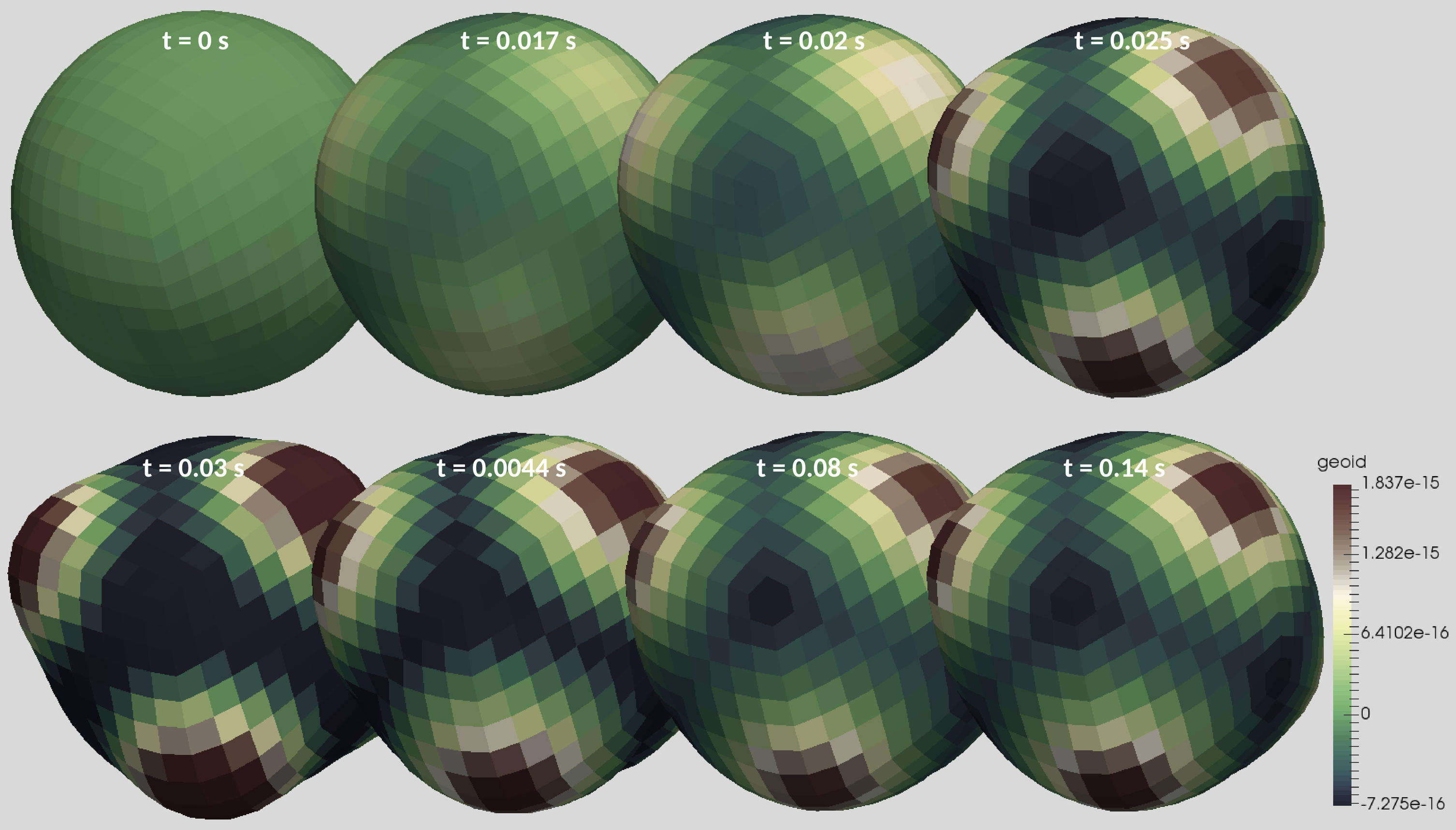
3 ASPECT – FREE SLIP:

Root-mean square velocity (Vrms), average temperature (<T>) and Nusselt number (Nu) sensitivities to
- grid resolution (level 3-4-5) and Courant number (CFL)
- stabilisation, with or without Boussinesq
- initial temperature perturbation (delta – ϵ in equation 8) – ***Similar tests to Arrial et al. 2014 using CitcomS*** - - - - - -





4 ASPECT – FREE SURFACE:



Initial Vrms (m/s) depending on model resolution and thermal expansion coefficient:

FREE SLIP (thermal expansion)

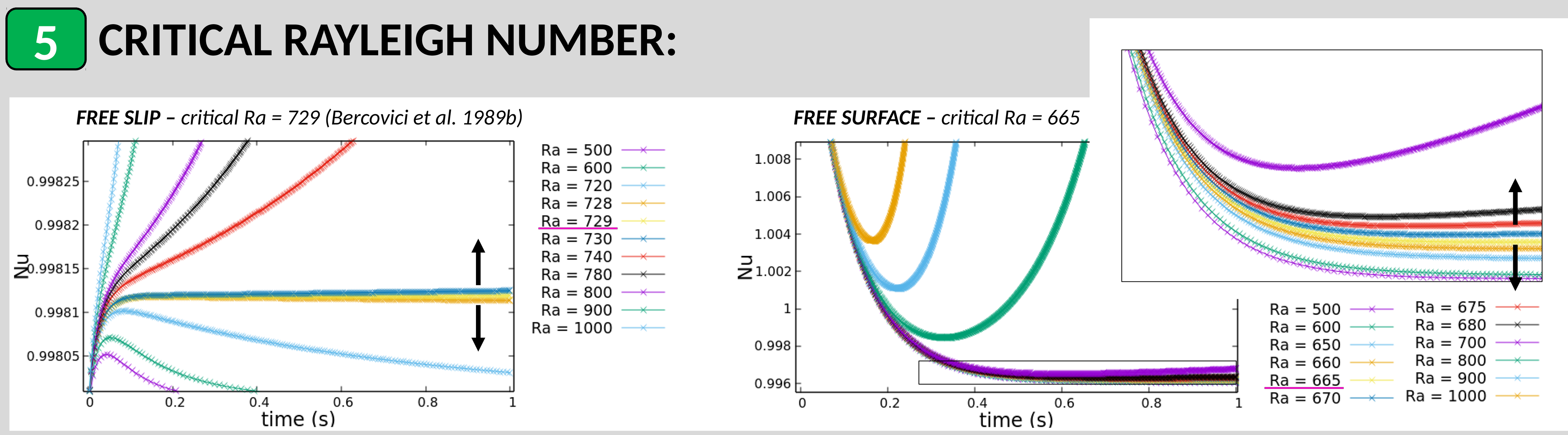
resolution	1	1e[-1]	1e[-2]	1e[-3]	1e[-4]	1e[-5]
level 3	0.647	0.655	0.981	7.01	69.5	694
level 4	0.647	0.647	0.652	0.88	5.72	56.6
level 5	0.647	0.647	0.647	0.65	0.815	4.8

FREE SURFACE (thermal expansion)

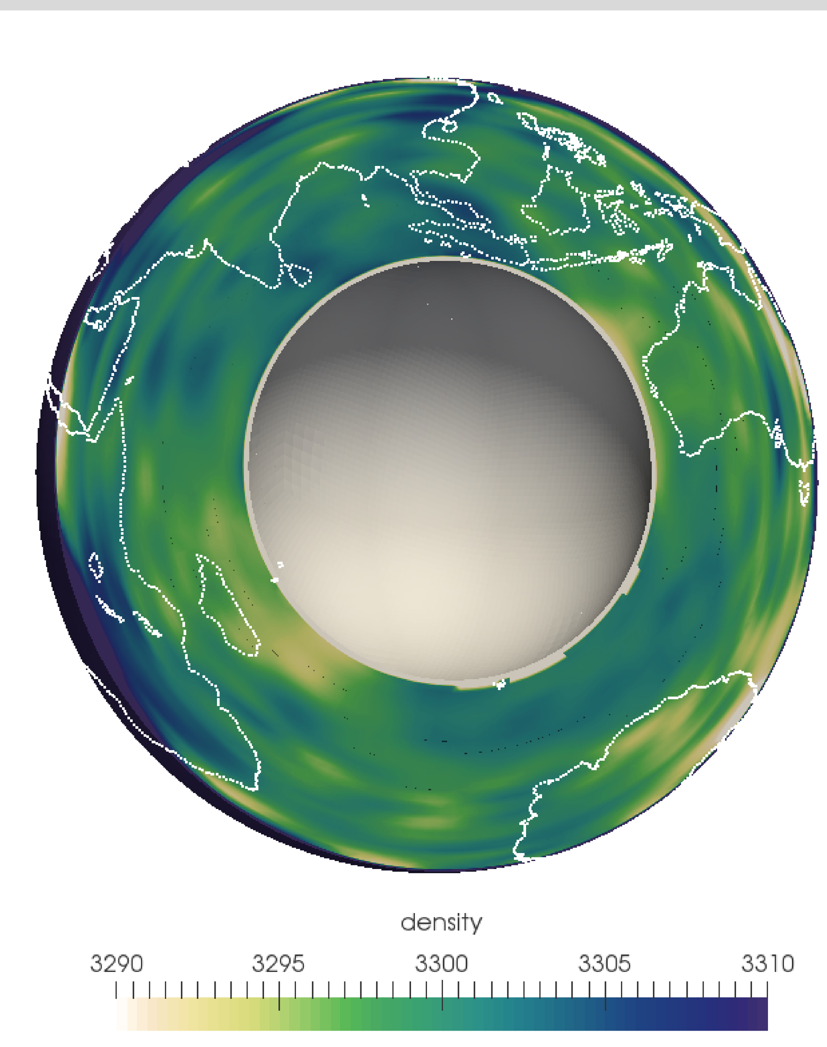
resolution	1	1e[-1]	1e[-2]	1e[-3]	1e[-4]	1e[-5]
level 3	1.89	3.01	27.9	283	2830	28300
level 4	1.93	1.91	7	71.1	715	7160
level 5	1.94	1.91	2.32	17.6	179	1790

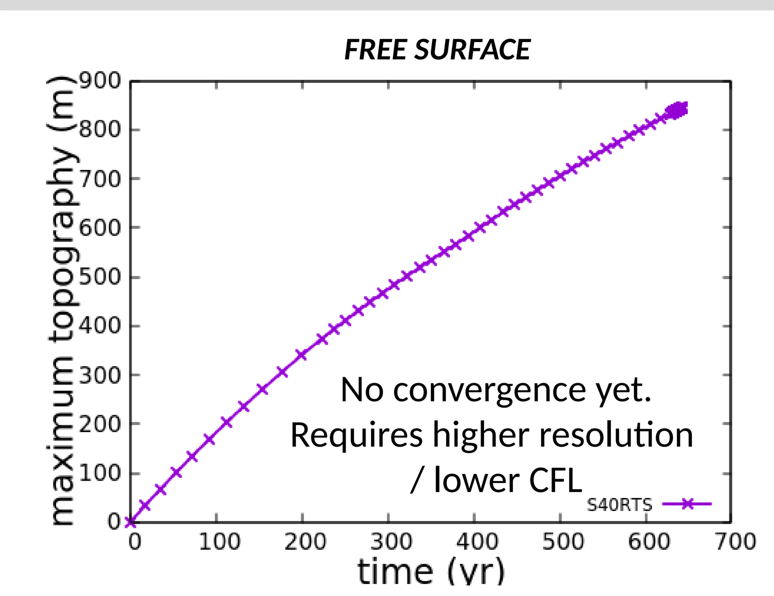
We find that using a free surface top boundary, the convection pattern is altered and more sensitive to model resolution.

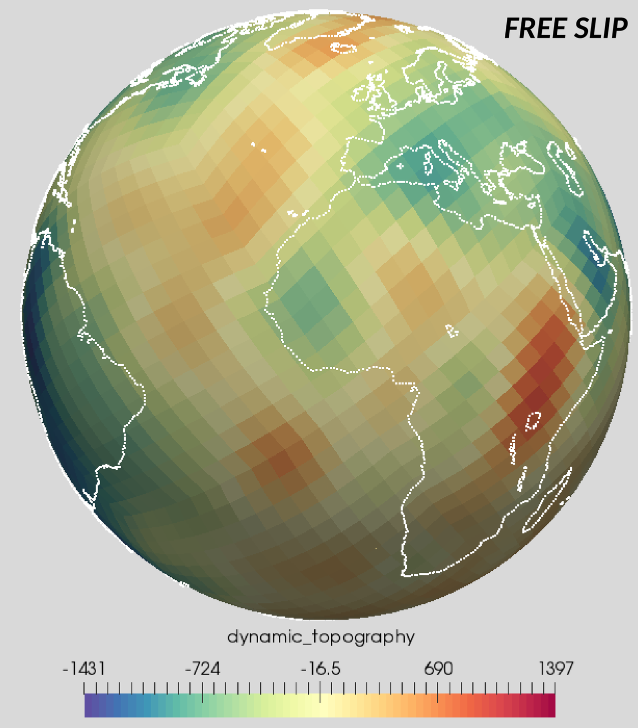
5 CRITICAL RAYLEIGH NUMBER:



6 PERSPECTIVES – WORK IN PROGRESS: Using S40RTS data







We incorporate real density in a spherical shell with a free surface and compare model prediction on dynamic topography (free slip) with topography (free surface)

Conclusion:
- Numerical experiments and sensitivity test with ASPECT using free slip vs free surface
- Critical Rayleigh number of free surface is slightly less than for free slip
- Predictions of similar topography to real Earth require an adequate choice of resolution and thermal expansion coefficient.