EFFECTS OF APERTURE ON MARCHENKO FOCUSING FUNCTIONS AND THEIR RADIATION DIRECTION

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SUMMARY

A focusing function is a specially constructed field that focuses onto a purely downgoing pulse at a specified subsurface position upon back propagation (injection) into the medium. Such focusing functions are key ingredients in the Marchenko method and in its applications such as retrieving Green's functions, redatuming, imaging with multiples, and synthesizing the response of virtual sources/receiver arrays at depth. In this study, we show how the focusing function and its corresponding focused response at a specified subsurface position are heavily influenced by the data aperture at the surface. We describe such effects by considering focusing functions in the context of time-domain imaging, offering for the first time explicit connections between time processing and Marchenko focusing. In particular, we show that the focused response radiates in the direction perpendicular to the line drawn from the center of the surface data aperture to the focused position in the time-imaging domain, i.e., in time-migration coordinates. The corresponding direction in the Cartesian domain follows from the combination of the time-domain direction and the directional change due to time-to-depth conversion. Therefore, the result from this study provides a better understanding of focusing functions and has implications in applications such as the construction of amplitude-preserving redatuming and imaging, where the directional dependence of the focused response plays a key role in controlling amplitude distortions.

FOCUSING IN TIME AND DEPTH DOMAINS

The downgoing focusing function f_1^+ is defined as the inverse of transmission operator T, which can be written as (left) or in the Fourier domain at some angular frequency (right)

$$\delta(\mathbf{x}-\mathbf{x}_f)\delta(t) = \int_{S_a} T(\mathbf{x}_f,\mathbf{x}'_a,t) * f_1^+(\mathbf{x}'_a,\mathbf{x}_f,t)\ d\mathbf{x}'_a \qquad \text{or} \qquad \mathbf{I} = \mathbf{T}\mathbf{F}_1^+$$

* denotes time convolution and $\mathbf{x}$ denotes different positions on the acquisition surface S_a and the focusing surface S_f. In the Fourier domian, $\mathbf{T}$ denotes a $n_f \times n_a$ transmission matrix for the truncated medium with elements representing the Green's functions between n_a points on S_a and n_f focused points on S_f at some specified depth. We seek $\mathbf{F}_1^+$ denoting a discrete $n_a \times n_f$ downward-going focusing function that is an inverse of $\mathbf{T}$. When the medium is smooth, there is only one event in f_1^+ associated with the direct waves traveling from the specified focusing position to the surface. On the other hand, if the medium produces scattering (i.e., reflections, diffractions), when injecting the downgoing focusing function f_1^+, signals are reflected upward on its path to generate a focused field at some specified location. This upward reflected response is referred to as f_1^-.

To create a focused field in the subsurface, Wapenaar et al. (2014) show that one needs to back propagate (inject) the total downgoing field f at the surface given by:

$$f(\mathbf{x}_f,\mathbf{x}_a,t) = f_1^+(\mathbf{x}_a,\mathbf{x}_f,t) - f_1^-(\mathbf{x}_a,\mathbf{x}_f,-t)$$

where we emphasize that f is to be injected from the surface position $\mathbf{x}_a$. The f_1^- is now time-reversed and subtracted from f_1^+ to handle, in real time, the unwanted reflections from the injection of f_1^+.

Upon the retrieval of focusing function f from the Marchenko method, we employ the acoustic wave equation and its counterpart in the time-imaging domain (Fomel, 2013) to back propagate them into the subsurface model. Their expressions can be given as follows:

Depth (Cartesian) coordinates

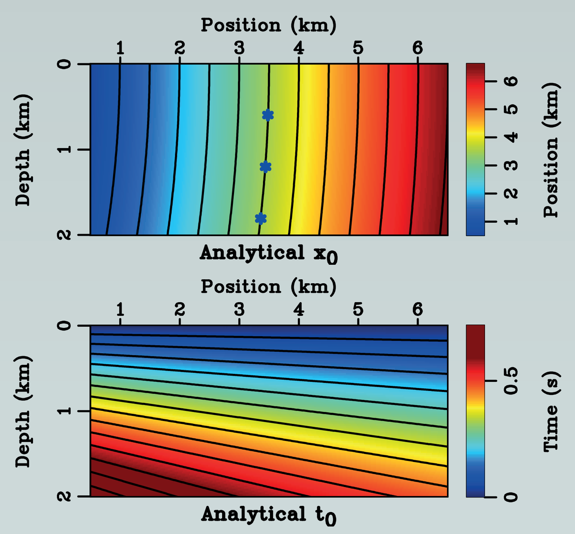
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{v^2(x,z)}\frac{\partial^2 u}{\partial t^2}$$

Time (image-ray) coordinates

$$v_d^2(x_0,t_0)\frac{\partial^2 u}{\partial x_0^2} + \frac{\partial^2 u}{\partial t_0^2} = \frac{\partial^2 u}{\partial t^2}$$

We use u to denote the wavefield, v is the medium velocity in the Cartesian coordinates, and v_d is the Dix velocity obtained after applying the Dix inversion on time-migration velocity. The Dix velocity v_d lives in the image-ray coordinates defined by x_0 and t_0 for the surface escape location and the one-way traveltime of image rays, respectively. We will use both wave equations to back propagate the focusing function injected at the surface, and establish connections between the directionally-dependent properties of the focused responses in both domains. In the beginning, we consider the case of smooth-background media with $f_1^- = 0$ and the initial f_1^+ approximated by the time-reversed direct-wave Green's function, which is taken as a leading-order estimate of the time-reversed transmission response.

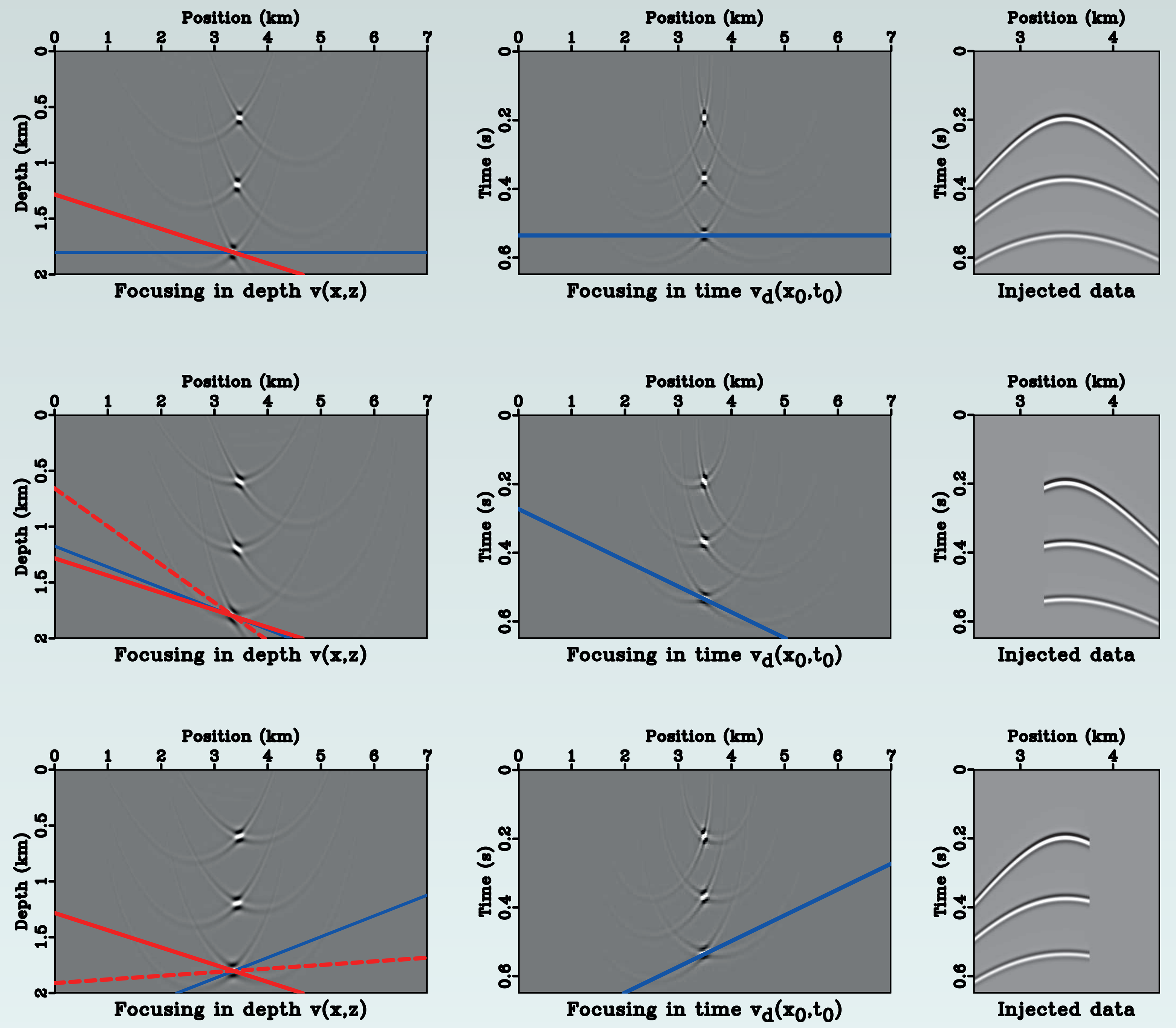
DIRECT-WAVEFIELD RADIATION AT DEPTH



We consider a synthetic medium with constant velocity gradients in both $\mathbf{x}$ and $\mathbf{z}$ directions with analytical image-ray map given by

$$v(x,z) = 2.0 + 0.6\ z + 0.25\ x$$

We look at three scatterers along the image rays originating from 3.5 km as noted by the stars in the Figure to the left. According to the relationship between the two coordinates as defined by image rays, we expect the focused responses to align along the vertical in the time coordinates and follow the trajectory of the image ray in the depth coordinates. The results from back propagation are shown below, where we note three-color lines. The solid red line denotes the image wavefront at the focused position, i.e, a surface of constant t_0. On the other hand, the solid blue line is perpendicular to the line drawn from the middle of the receiver array (injecting sources for reverse-time propagation) to the focused position in the time domain. The dashed red line that has a combined slope of the solid red and blue lines.



From this experiment, we can make three important observations:

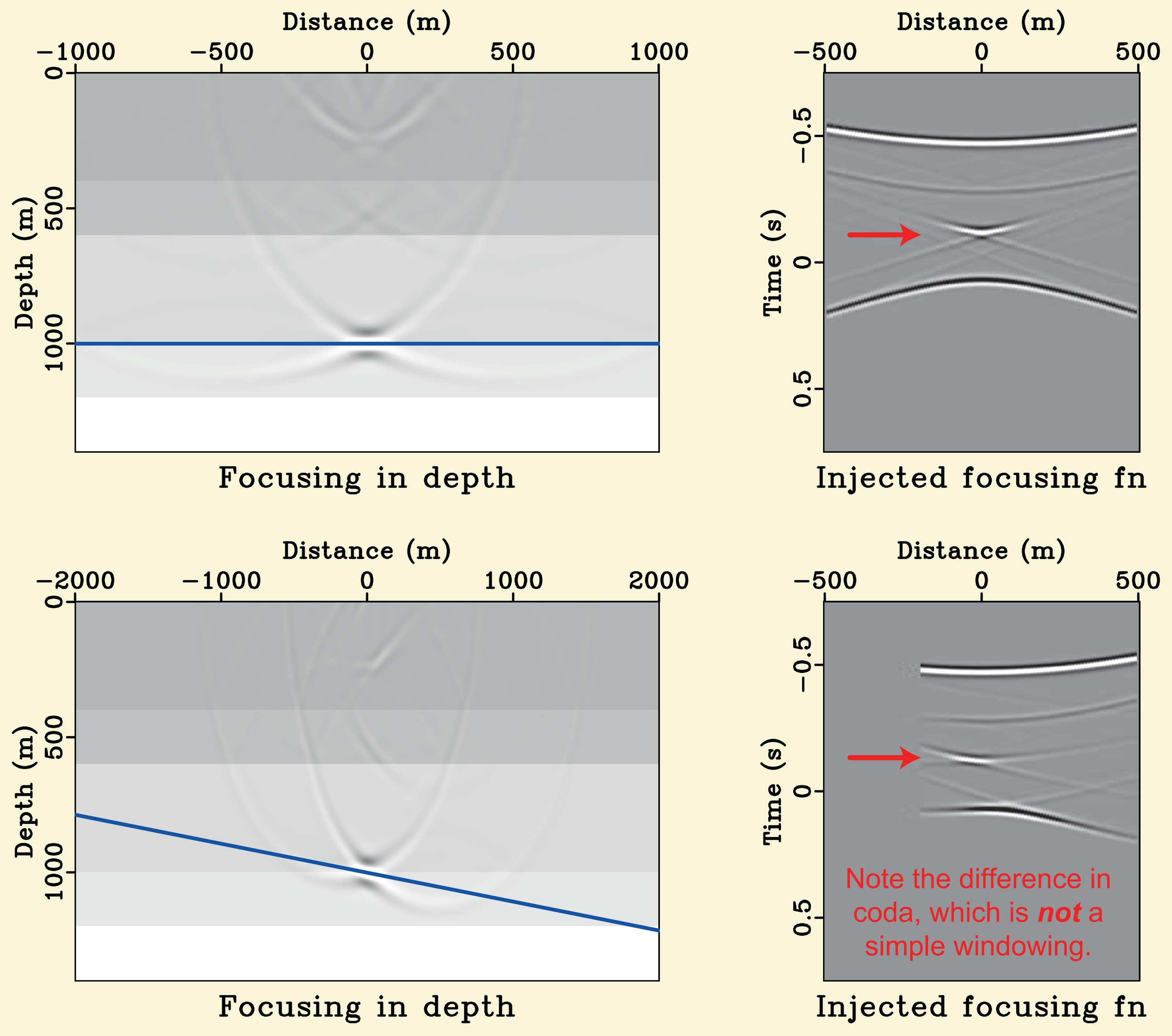
1) Having a receiver array that is asymmetric relative to the focal position (i.e., limiting data aperture) is equivalent to windowing/weighting parts of the data, changing the dynamics (amplitudes) of the resulting focused field but not the location of the focal points. Because the focused positions remain the same, they are controlled solely by the kinematics (phase/traveltime) of the available data.
2) Under time-domain imaging, the directionality of the focused response (blue) is perpendicular to the line drawn from the middle of the surface receiver array to the focused position in the time (image-ray) coordinates.
3) In the depth (Cartesian) coordinates, the directionality of the focused response is a direct combination (dashed red) of the effects from limited aperture (blue) and the bending of image rays (red) used for time-to-depth conversion.

MARCHENKO FOCUSING FUNCTIONS RADIATION DIRECTION

1D model

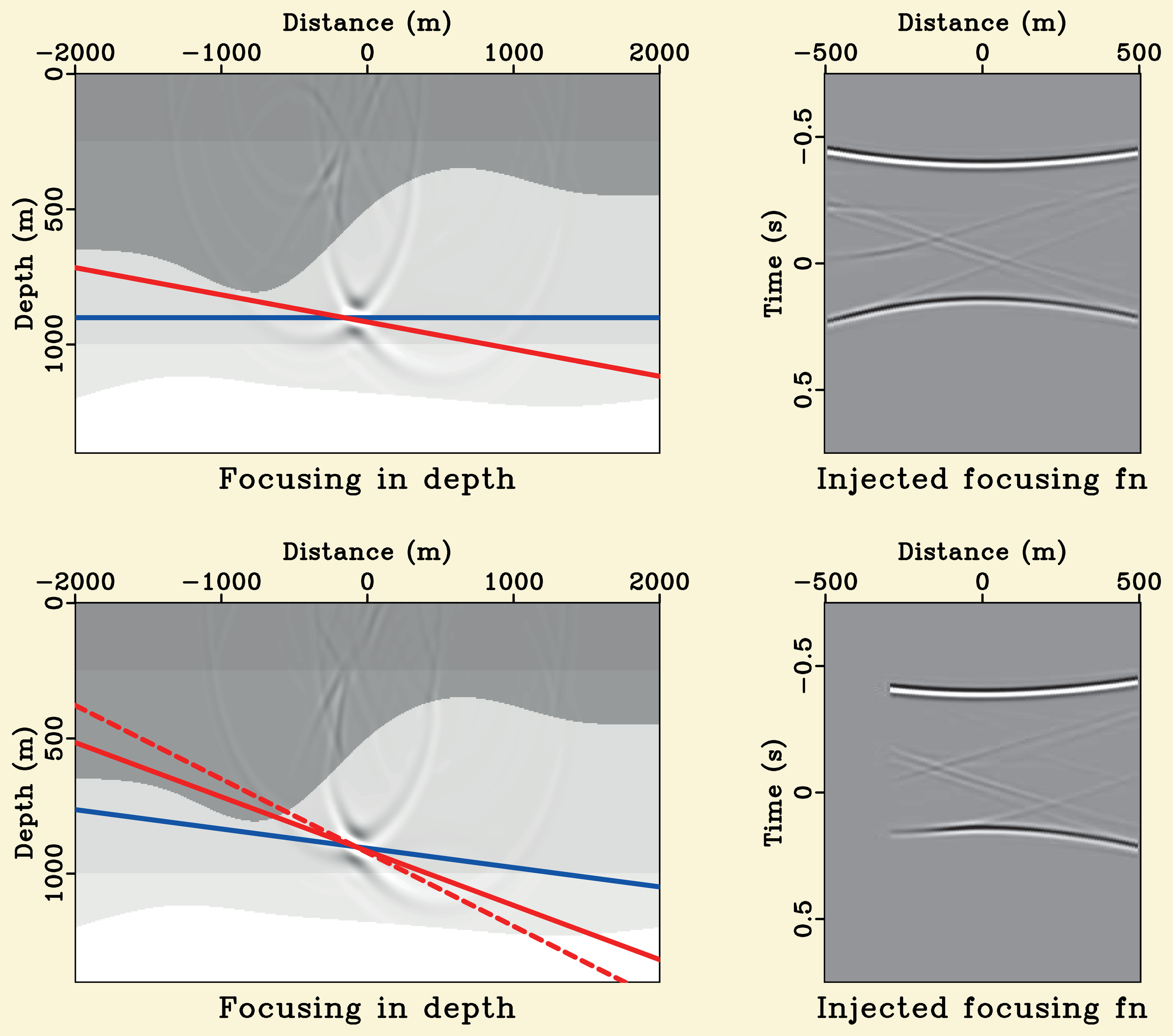
We consider a focal point at (0,1000) on the third reflector. The image rays are vertical in this model and the additional change of the radiation direction due to the time-to-depth conversion can thus be neglected. We follow the same procedure as before and the focused responses are at $t = 0$ shown on the right for both symmetric and asymmetric aperture cases.

These results are not entirely surprising because, given the choice of a direct-wave Green's function with an arbitrary spatially-dependent weighting (e.g, limited aperture), the Marchenko method solves for f that in turn would lead to a focused response that is directionally-weighted according to such a choice. Therefore, the observations on the effects of aperture on the focused response of direct waves can be extended to the case of any version of any spatially-weighted Marchenko focusing functions.



2D model

Next, we turn to a 2D laterally heterogeneous model with bending image rays, where both the effects from asymmetric surface aperture and the time-to-depth conversion are important. We consider the focusing position at (-80, 900), which is along the image ray originating from 0 m. The resutls are shown on the right. We can see that the directionality of the focused response is no longer defined by the solid blue line but instead by the solid red (symmetric) and the dashed red (asymmetric). The observed results agree with those from the direct waves and the 1D model. We also note the observed artifacts are due to the small surface aperture (-500, 500) and (-300, 500) used.



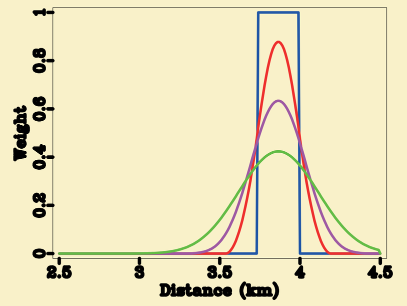
BLURRING-BASED INTERPRETATION

An interpretation of such observations can be achieved by considering a least-squares estimate of $\mathbf{F}_1^+$

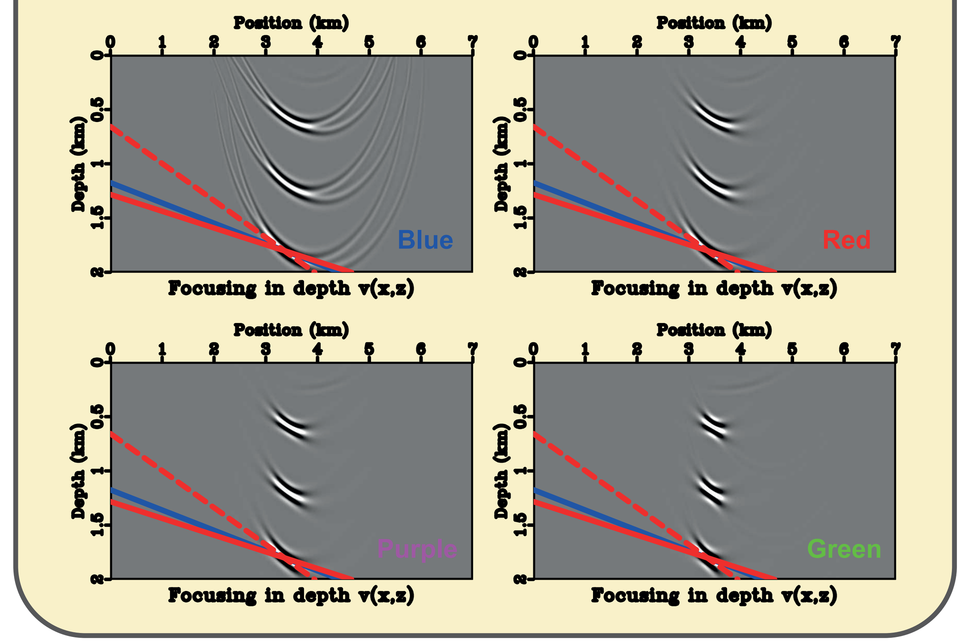
$$(\mathbf{F}_1^+)_{\text{LS}} = (\mathbf{T}^\dagger\mathbf{T})^{-1}\mathbf{T}^\dagger = \mathbf{D}\mathbf{T}^\dagger$$

where $\mathbf{D}$ is a deblurring operator. In other words, a focusing function that will focus at a specified position can be interpreted as a deblurred version of the adjoint (time-reversed) transmission response as used in this experiment. Because $\mathbf{D}$ is zero-phase (being the inverse of the operator $\mathbf{T}^\dagger\mathbf{T}$), the amplitude weight of the surface data (Heaviside weighting in our case) controls the directionality of the focused response in both time and depth domains, while not altering the location of the focal point. A mathematically extensive treatment of directionally-varying `amplitude blurring', in the context of depth imaging, is presented by Thomson et al (2016).

SPATIALLY-VARYING WEIGHT



We also experiment with edge-tapered weights and observe that the artifacts are reduced, but our observations on the directionality effects of this weighting remain applicable.



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