Early Explorations in Benchmarking Thermal Convection in a Two-Dimensional Spherical Annulus

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Abstract and Introduction

Through understanding the differences and similarities of mantle convection on the earth and other planets, we can get a better understanding of processes like thermal evolution and plate tectonics. Mantle convection is a long term process and happens deep inside planets. Therefore, modeling mantle convection is currently the best way to get an understanding of the dynamics of planetary interiors. We present results of dimensionless convection experiments in two-dimensional spherical domains obtained using both ASPECT[2, 3] and ELEFANT [4, 5, 6] geodynamical codes. The domain is scaled such that the mantle thickness and the temperature difference between the inner and outer boundaries are one. The initial temperature profile is linear with a superimposed spherical harmonic perturbation of small amplitude. A temperature-dependent rheology type earlier used in Tosi et al. [1] was used. We ran a series of experiments in which the order of the initial spherical harmonic perturbation was varied between three and eleven. The temperature perturbations initially give rise to the same amount of convection cells. These undergo a series of spatial reorganizations with decreasing cell numbers, ultimately reaching a statistical steady state corresponding to a constant cell number which is smaller than the number of initial perturbations. Current results indicate that the final number of convection cells would be 3. Here, we specifically look at the time evolution values of the V_{rms}, the volume averaged temperature, the Nusselt number and the temperature field power spectrum. We analyze various sets of experiments and show how numerical parameters affect the results for both codes.

Setup

Geometry and Benchmark

We use a two-dimensional annulus domain. The inner radius and outer radius are scaled to the earth's mantle such that the mantle thickness is 1. The temperature field is described as follows:

$$T(s,\theta) = s + As(1-s)\cos(N_0\theta) \tag{1}$$

where N_0 is the number of initial perturbations we use to initiate plume formation, A is the amplitude of these perturbation and s is the depth.

Model Parameters

Parameter Name	Symbol	Value
Inner radius	R_{inner}	1.22
Outer radius	R_{outer}	2.22
Inner temperature	T_{inner}	1
Outer temperature	T_{outer}	0
Reference temperature	T_0	0
Reference density	ρ_0	1
Reference viscosity	η_0	10^{-5}
Thermal viscosity parameter	$\Delta\eta_T$	10^5
Minimum viscosity	η_{min}	10^{-5}
Maximum viscosity	η_{max}	1
Thermal conductivity	k	1
Thermal expansivity	α	10^{-2}

Conservation Equations and Rheology

We solve the following equations for an incompressible fluid, under the Boussinesq approximation:

$$\nabla\cdot\mathbf{u} = 0 \tag{2}$$

$$\nabla\tau - \nabla p = \rho\mathbf{g} \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T\right) = \nabla\cdot k\nabla T \tag{4}$$

$$\rho = \rho_0(1-\alpha(T-T_0)) \tag{5}$$

$$\eta(T) = \exp(-\gamma_T T),\ \gamma_T = \ln\Delta\eta_T \tag{6}$$

Measurements

We calculate the following output parameters: the average temperature $< T >$ (Eq. 7), the root mean square velocity V_{RMS} (Eq. 8), the heat transfer rate through both boundaries Q (Eq. 9) and the Nusselt number at both boundaries Nu (Eq. 10).

$$\langle T\rangle = \frac{\int_\Omega T(r,\theta)d\Omega}{\int_\Omega d\Omega} \tag{7}$$

$$V_{RMS} = \left(\frac{\int_\Omega (v_r^2+v_\theta^2)\,d\Omega}{\int_\Omega d\Omega}\right)^{1/2} \tag{8}$$

$$Q_{inner,outer} = R_{inner,outer}\int_\Gamma \boldsymbol{q}\cdot\boldsymbol{n}\,d\Gamma \tag{9}$$

$$Nu_{inner,outer} = Q_{inner,outer}\Bigg/\frac{-k\,2\pi}{\ln\frac{R_{inner}}{R_{outer}}} \tag{10}$$

Furthermore, we implemented calculations for the power spectrum of the temperature field:

$$PS_n(T) = \left|\int_\Omega T(r,\theta)e^{in\theta}d\Omega\right|^2. \tag{11}$$

References

[1] Tosi et al. G3, 2015. [2] Kronbichler et al., GJI, 2012. [3] Heister et al. GJI, 2017. [4] Thieulot, Solid Earth, 2017. [5] Thieulot and Puckett, Comp. and Geosc., Subm. [6] Plunder et al., Solid Earth, 2018.

Influence of Input Parameters

We looked at the influence of the input parameters in both ASPECT and ELEFANT. The parameters and whether they influenced the results can be found in the table underneath.

Reference models	ASPECT	ELEFANT	Influence ASPECT	ELEFANT
Number of initial perturbations	11	11	next section	next section
Amplitude of perturbations	0.2	0.2	not tested	no (when A > 0.01)
CFL number	1.0	0.5	no	no
Nonlinear solver tolerance	10^{-5}	10^{-6}	no	not tested
Stokes velocity polynomial degree	2	-	yes	not tested
Locally conservative discretization	No	-	yes	not tested
Initial Global Refinement	5	-	yes	-
Resolution	-	48	-	yes
Stabilization	No	-	yes	-
Inner boundary	Free-slip	No-slip	yes	not tested
Outer boundary	Free-slip	Free-slip	not tested	not tested

Influence of the Initial Perturbations on the Steady State

We investigated the influence of the number of initial perturbations. We ran experiments in both ASPECT and ELEFANT with 3, 5, 7, 9 and 11 initial perturbations. Because, the nullspace removal type of algorithm influenced the output values we resorted to use no-slip conditions on the inner boundary.

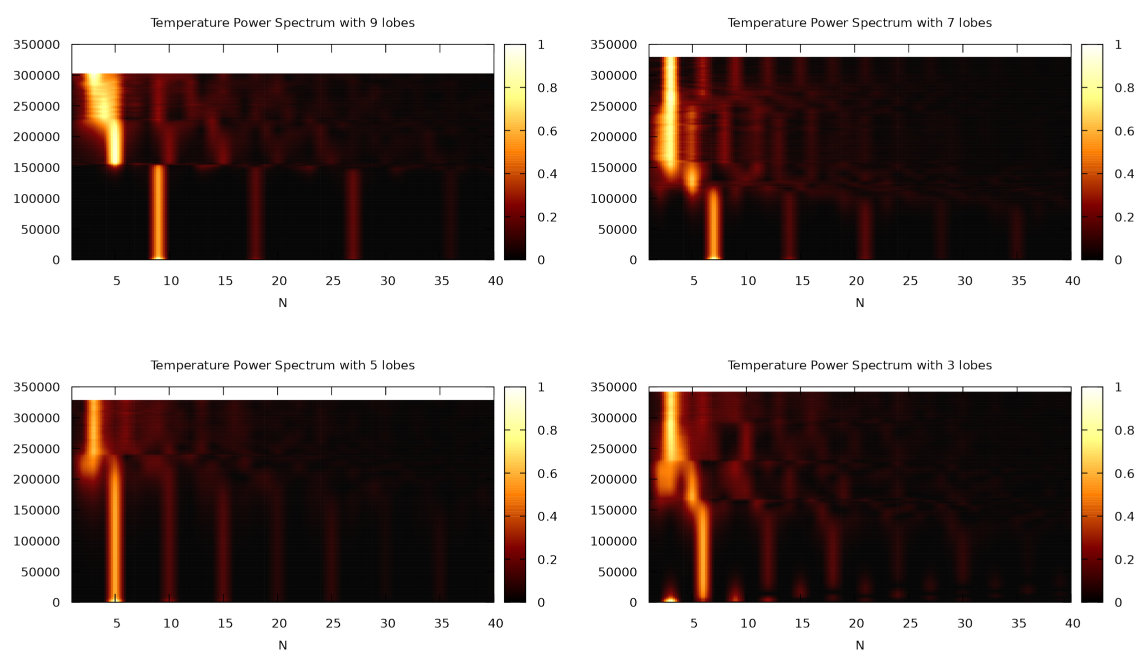

Figure 1. Temperature field power spectra of the simulations with respectively nine, seven, five and three initial perturbations.

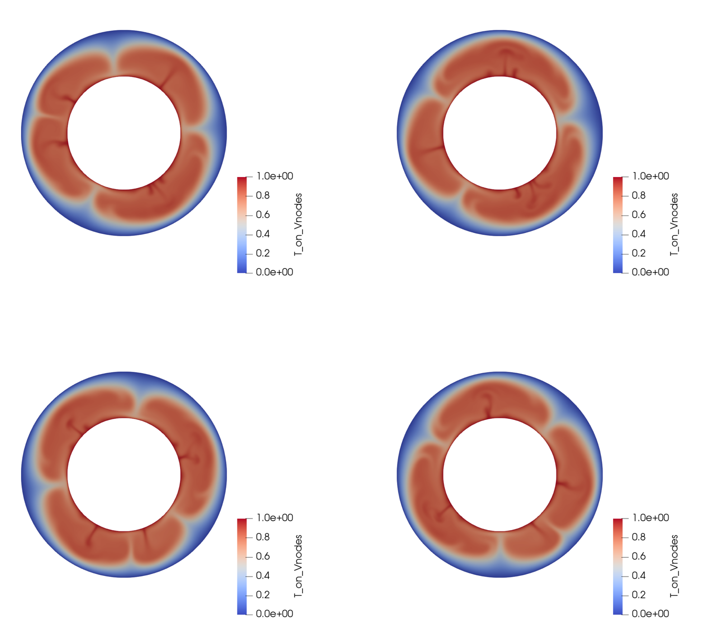

Figure 2. Snapshots of the end-state of the simulations with respectively nine, seven, five and three initial perturbations.

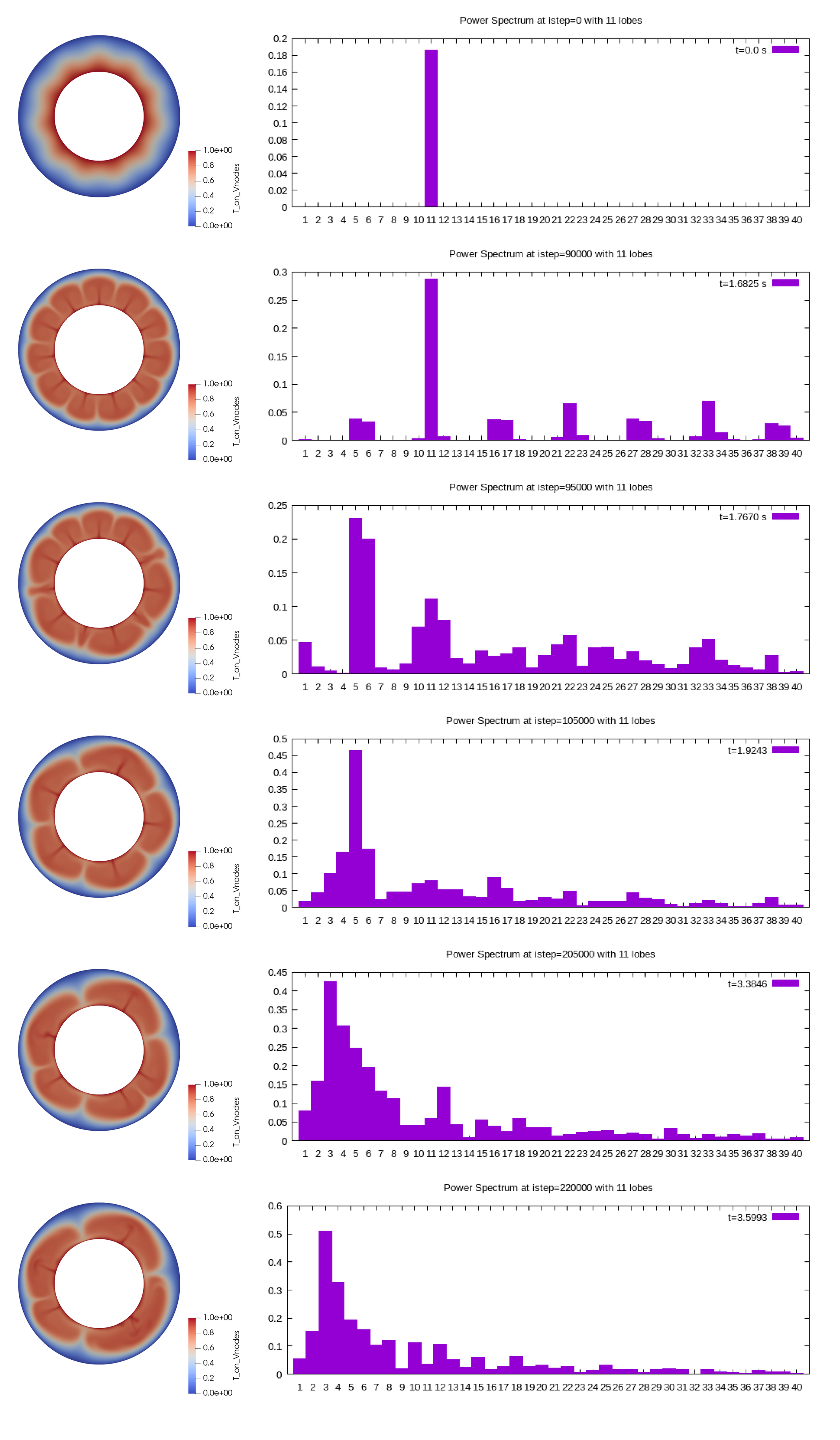

Figure 3. Snapshots of the temperature field and plots of the temperature field spectrum through time obtained using ELEFANT with eleven initial perturbations.

Conclusions and Future Work

Conclusions.

- All simulations reorganize to a statistical steady state with less convection cells.
- All simulations reorganize to a statistical steady state with three or four convection cells.
- A statistical steady state with a total of five or less convection cells shows time-dependent behavior.
- As a result of the time-dependent behavior the current setup is not ideal for doing a benchmark test.

Future work.

- Investigating other cases from Tosi et al. which include depth and strain-rate dependent rheologies.
- Analyzing the time-dependent behavior.
- Implementing the models in FIELDSTONE, a new finite element code with an educational purpose.
- Investigate the influence of other input parameters (e.g. Adaptive mesh refinement)