

Utrecht Consortium *for* Subsurface



Summary

The Marchenko method has been used for generating Green's functions and focusing functions for many years. This method requires that the sources and receivers of the reflection data should be co-located and densly sampled, which is hardly the case in real acquisitions Our purpose in this study here is to understand the effect of subsampling under the Marchenko framework. The subsampling problem exists in many cases, like in streamer or OBC data. In practice, one can use interpolation to fill the gaps in data. But this kind of method has its own drawbacks. Understanding the effect of subsampling can help us overcome this issue. In this study, we focus particularly on just sampling one dimension, source or receiver with the other dimension untouched.

Discrete integration in Marchenko framework

According to Wapenaar et al. (2014), the first equation of the coupled Marchenko system for the acoustic medium can be defined in frequency domain by integrating on the dipole-source dimension with the receiver being monopole:

$$\hat{G}^{-}(\mathbf{x}_{F};\mathbf{x}_{r}) = \int_{\Lambda_{f}} d\mathbf{x}_{s} \hat{R}(\mathbf{x}_{r};\mathbf{x}_{s}) \hat{f}_{1}^{+}(\mathbf{x}_{s};\mathbf{x}_{F}) - \hat{f}_{1}^{-}(\mathbf{x}_{r};\mathbf{x}_{F})$$

R is the discrete integral operator that applies multidimensional convolution between the reflection response and the focusing functions. We focus on the first operation Rf_{1d}^+ of the iterative substitution method in the form of matrix-vector multiplication (corresponding to the discrete spatial convolutions):

$$\begin{bmatrix} \hat{f}_{1,\mathcal{K}=0}^{-}(\mathbf{x}_{r}^{(0)};\mathbf{x}_{F}) \\ \mathbf{i} \\ \hat{f}_{1,\mathcal{K}=0}^{-}(\mathbf{x}_{r}^{(Nr)};\mathbf{x}_{F}) \end{bmatrix} \propto \begin{bmatrix} \sum_{i=1}^{N_{s}} \hat{R}(\mathbf{x}_{r}^{(0)};\mathbf{x}_{s}^{(i)}) \hat{f}_{1d}^{+}(\mathbf{x}_{s}^{(i)};\mathbf{x}_{F}) d\mathbf{x}_{s} \mathbf{x}_{s} \\ \sum_{i=1}^{N_{s}} \hat{R}(\mathbf{x}_{r}^{(Nr)};\mathbf{x}_{s}^{(i)}) \hat{f}_{1d}^{+}(\mathbf{x}_{s}^{(i)};\mathbf{x}_{F}) d\mathbf{x}_{s} \mathbf{x}_{s} \end{bmatrix}$$

The *j*th element on the left-hand-side corresponds to the trace at receiver position $\mathbf{x}_{r}^{(j)}$, that is calculated by summing the convolution gather between $\hat{R}(\mathbf{x}_{r}^{(j)};\mathbf{x}_{s})$ and $\hat{f}_{1d}^+(\mathbf{x}_s; \mathbf{x}_F)$ for all the source positions and scaled by the (possibly subsampled) source interval. By employing source-receiver reciprocity, the same Marchenko equation can also be defined by integrating on the dipole-receiver dimension with the source being a monopole (Van der Neut et al., 2015):

$$\hat{G}^{-}(\mathbf{x}_{F};\mathbf{x}_{s}) = \int_{\Lambda_{f}} d\mathbf{x}_{r} \hat{R}(\mathbf{x}_{r};\mathbf{x}_{s}) \hat{f}_{1}^{+}(\mathbf{x}_{r};\mathbf{x}_{F}) - \hat{f}_{1}^{-}(\mathbf{x}_{s};\mathbf{x}_{F})$$

The initial-iteration discrete convolution has the form:

$$\begin{bmatrix} \hat{f}_{1,\mathcal{K}=0}^{-}(\mathbf{x}_{s}^{(0)};\mathbf{x}_{F}) \\ \mathbf{i} \\ \hat{f}_{1,\mathcal{K}=0}^{-}(\mathbf{x}_{s}^{(Ns)};\mathbf{x}_{F}) \end{bmatrix} \propto \begin{bmatrix} \sum_{i=1}^{N_{r}} \hat{R}(\mathbf{x}_{r}^{(i)};\mathbf{x}_{s}^{(0)}) \hat{f}_{1d}^{+}(\mathbf{x}_{r}^{(i)};\mathbf{x}_{F}) d\mathbf{x}_{r} \\ \sum_{i=1}^{N_{r}} \hat{R}(\mathbf{x}_{r}^{(i)};\mathbf{x}_{s}^{(Ns)}) \hat{f}_{1d}^{+}(\mathbf{x}_{r}^{(i)};\mathbf{x}_{F}) d\mathbf{x}_{r} \end{bmatrix}$$

When integrating over the receiver locations, the *j*th-element output in the obtained upgoing focusing function now corresponds to a trace at source position $x_s^{(j)}$, that is calculated by integrating the convolution gather $\hat{R}(\mathbf{x}_r; \mathbf{x}_s^{(j)})$ and $\hat{f}_{1d}^+(\mathbf{x}_r; \mathbf{x}_F)$ over the receiver positions, scaled by the (possibly subsampled) receiver interval. Here, we rely on these two matrix-vector multiplication forms to interpret the observed effects of subsampling over source and receiver dimensions.

Effects of acquisition sampling on 2D Marchenko focusing and redatuming

Joint effect of subsampling vs integration variable choice in multidimensional convolution

every 8 sources or receivers.

Figure 2: Comparison of focusing functions for the layered model of different subsampling and integration choices in time domain. The red, blue and black traces show the zoomed-in plot for different focusing functions (no subsampling, with source subsampling while integrating on source, with receiver subsampling while integrating on source) at far offset $(X_r = -910m)$, respectively.

The color bars denote the corresponding normalized errors.

Bibliography

Wapenaar, K., Thorbecke, J., Van der Neut, J., Broggini, F., Slob, E., and Snieder, R. [2014] Green's function retrieval Van der Neut, J., Vasconcelos, I., and Wapenaar, K. [2015] On Green's function retrieval by iterative substitution of from reflection data, in absence of a receiver at the virtual source position. Journal of the Acoustical Society of the coupled Marchenko equations. Geophysical Journal International, 203, 792-813. America, 135(5), 2847-2861.

Figure 4: Summary of subsampling vs integration variable choice. Subsampling and integrating on the same dimension will yield focusing functions with artifacts but no spatial gaps. Subsampling and integrating on different dimensions will yield focusing functions with artifacts together with spatial gaps. Yellow arrows denote the integration dimension. Grey squares represent the missing data.

Figure 5: For the layered model, the integrand gather of Rf_{1d}^+ with receiver position $X_r = -40m$, $X_r = -400m$, $X_r = -960m$. The blue and red lines represent the results calculated with and without subsampling, respectively. The Magenta arrow and yellow line denote the stationary points and the slops of events, respectively.

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