The Marchenko method has been used for generating Green’s functions and focusing functions for many years. This method requires that the source and receivers of the reflection data should be located and densely sampled, which is hardly the case in real acquisitions. Our purpose in this study here is to understand the effect of subsampling under the Marchenko framework. The subsampling problem exists in many cases, like in streamer or OBC data. In practice, one can use interpolation to fill the gaps in data. But this kind of method has its own drawbacks. Understanding the effect of subsampling can help us overcome this issue. In this study, we focus particularly on just sampling one dimension, source or receiver with the other dimension untouched.

### Discrete integration in Marchenko framework

According to Wapenaar et al. (2014), the first equation of the coupled Marchenko system for the acoustic medium can be defined in frequency domain by integrating on the dipole-source dimension with the receiver being monopole:

\[
\hat{G}(x; s) = \int dx R(x; s) F(x; s) = \hat{f}(x; s)
\]

\(R\) is the discrete integral operator that applies multidimensional convolution between the reflection response and the focusing functions. We focus on the first operation of subsampling over source and receiver dimensions.

The Marchenko method has been used for generating Green’s functions and focusing functions. We focus on the first operation \(R\) of the iterative substitution method in the form of matrix-vector multiplication (corresponding to the discrete spatial convolutions):

\[
\hat{f}_{r[i]} = R_{r[i]}(\hat{s}; x)\hat{f}_{s[i]}
\]

The \(i\)th element on the left-hand-side corresponds to the trace at receiver position \(x_i\), that is calculated by summing the convolution gather between \(R(\hat{s}; x)\) and \(\hat{f}_{s[i]}(x)\) for all the source positions and scaled by the (possibly subsampled) source interval. By employing source-receiver reciprocity, the same Marchenko equation can also be derived by integrating on the dipole-receiver dimension with the source being a monopole (Van der Neut et al., 2015):

\[
\hat{G}(s; x) = \int ds R(s; x) \hat{f}_{r[i]}(s; x) = \hat{f}(s; x)
\]

The initial-iteration discrete convolution has the form:

\[
\hat{f}_{r[i]}(s; x) = \int ds R(s; x) \hat{f}_{s[i]}(s; x) = \hat{f}(s; x)
\]

When integrating over the receiver locations, the \(j\)th element in the obtained focusing function now corresponds to a trace at source position \(x_j\) that is calculated by integrating the convolution gather \(R(s; x)\) and \(\hat{f}_{s[j]}(s; x)\) over the receiver positions, scaled by the (possibly subsampled) receiver interval. Here, we rely on these two matrix-vector multiplication forms to interpret the observed effects of subsampling over source and receiver dimensions.

### Summary

The Marchenko method has been used for generating Green’s functions and focusing functions for many years. This method requires that the source and receivers of the reflection data should be located and densely sampled, which is hardly the case in real acquisitions. Our purpose in this study here is to understand the effect of subsampling under the Marchenko framework. The subsampling problem exists in many cases, like in streamer or OBC data. In practice, one can use interpolation to fill the gaps in data. But this kind of method has its own drawbacks. Understanding the effect of subsampling can help us overcome this issue. In this study, we focus particularly on just sampling one dimension, source or receiver with the other dimension untouched.