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Near-well analysis of stresses utilizing a fast coupled hydro-mechanical poro-elastoplastic model

Peter A. Fokker[1,2], Aditya Singh[1]

1 HPT lab, Utrecht University, Utrecht, The Netherlands
2 TNO applied geoscience, Utrecht, The Netherlands

Introduction

The involved complexities in operating a wellbore for geothermal processes [see **Fig. 1**] are due to fact that this geotechnical structure has to perform under the constraints caused by fluid flow, complex mechanical response of the geo-materials and mechanical effects caused by the thermal perturbations due to injection. Over and above there are also the chemical effects on the mechanical responses due to chemically active geo-mass.

These processes do not only effect the integrity of the wellbore but also induced seismicity, by altering the virgin in-situ stress state. To assess the behavioral response of wellbore and near region under these complex operational and geological conditions, many rigorous numerical tools are been developed. They are good for the deterministic approach but are not advisable for probabilistic approach. This is because they are not efficient. On the other hand analytical solutions are very efficient but they are far too much idealistic. The fast semi analytical models could be the solution to the problem. They can handle more complexities then an analytical model and can be efficient.

In the current work a semi analytical approach is presented, which can handle the transient nature of the problem with reservoir stimulation and the progressive failure within the assumptions of asymmetry, homogeneity and plane-strain.

Modelling approach

Due to the construction of the wellbore, there will be formation of two types of regions. One will be a poro-elastic region, where the stresses will be lower then the yield surface of the rock mass; in a poro-elastoplastic region, stresses will be at the yield surface of the rock mass.

Poro-elastic region

For a material that is linear elastic, two parameters, shear modulus G and Poisson's ratio ν are used to define its behavior. **Detournay and Cheng (1988)** defined the relationships between induced total stress σ_{ij}, strain ε_{ij}, and pore pressure increase ΔP:

$$\sigma_{ij} = 2G\left[\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon\delta_{ij}\right] - \alpha\Delta P\delta_{ij} + \sigma_{ij}^{\infty}$$

Stress equilibrium in the horizontal plane for asymmetric condition is given in polar coordinates as:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} = 0$$

Using these fundamental equations the incremental solutions is given as:

$$u_r^{i+1} = u_r^i + \frac{1}{r\cdot 2G}\left\{\Delta I_{PT}^{i+1} + Z_1^{i+1} + \frac{1}{2}Z_2^{i+1}r^2\right\}$$

$$\sigma_{rr}^{i+1} = \sigma_{rr}^i - \frac{1}{r^2}\left[\Delta I_{PT}^{i+1}(r,t) + Z_1^{i+1}\right] + \frac{Z_2^{i+1}}{2(1-2\nu)}$$

$$\sigma_{\theta\theta}^{i+1} = \sigma_{\theta\theta}^i - \frac{1-2\nu}{1-\nu}\cdot\alpha\Delta P^{i+1} + \frac{1}{r^2}\left[\Delta I_{PT}^{i+1}(r,t) + Z_1^{i+1}\right] + \frac{Z_2^{i+1}}{2(1-2\nu)}$$

$$\Delta I_{PT}^{i+1}(r,t) = \frac{1-2\nu}{1-\nu}\cdot\int_{r_w}^{r}\rho\cdot\alpha\Delta P^{i+1}d\rho$$

$$\sigma_{zz}^{i+1} = \sigma_{zz}^i - \frac{1-2\nu}{1-\nu}\cdot\alpha\Delta P^{i+1} + \frac{Z_2^{i+1}\nu}{1-2\nu}$$

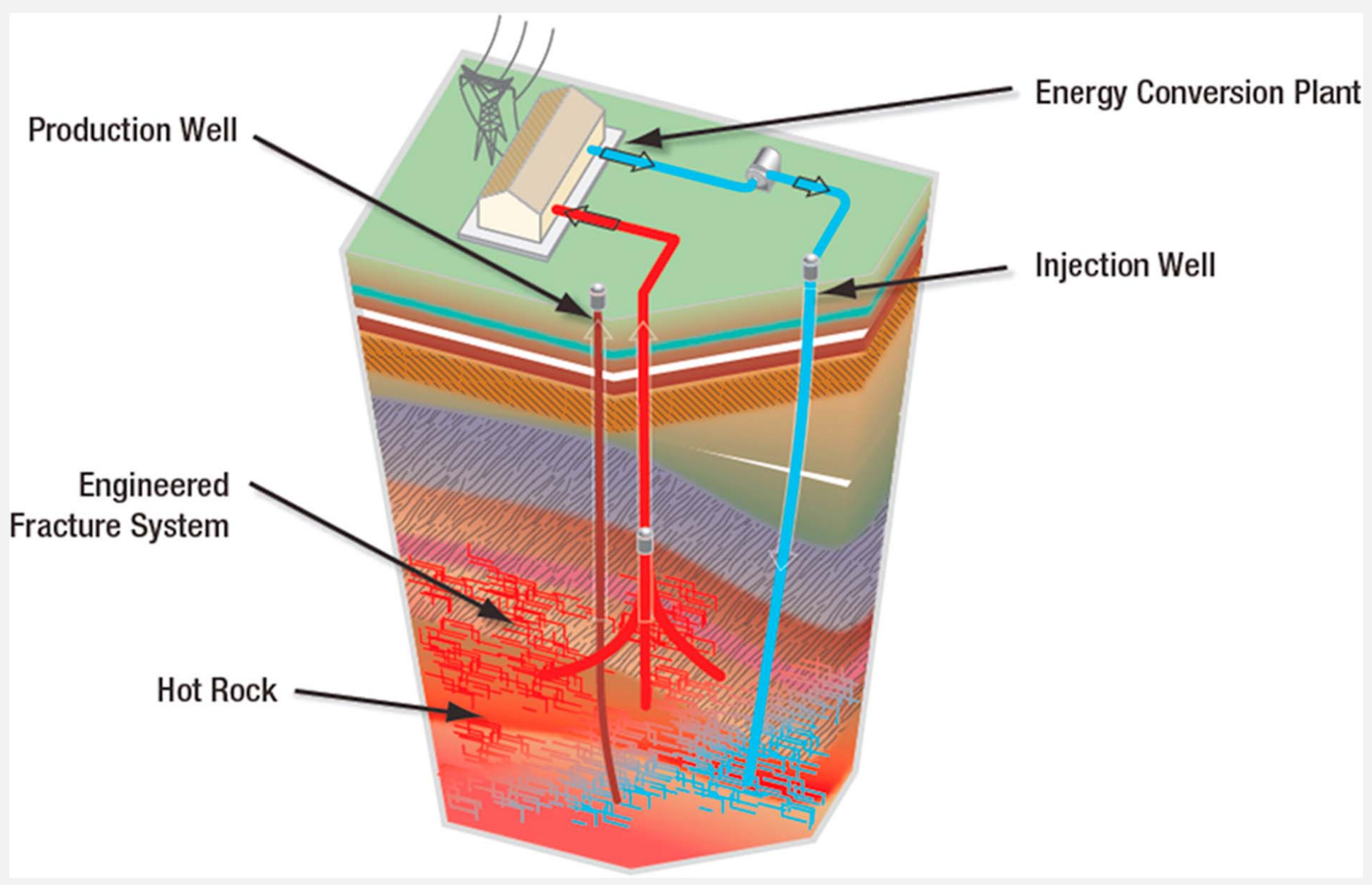

Figure 1
Schematic representation of the injection and production processes to assess the geothermal energy.

For a fluid flow, if large enough time is considered then an equivalent solution can be formulated as a logarithmic decay up to an influence radius – the pressure drop within that radius can thus be calculated using Darcy's law under the assumption of negligible storage (Grant, 2011).

$$\frac{dp}{dr} = -\frac{q(r)}{\lambda} = \frac{-\tilde{Q}}{2\lambda r} \text{ for } r < r_e(t)$$

where $\lambda = k/\mu$, the ratio of permeability and viscosity. r_e is calculated from the diffusion velocity.

Poro-elastoplastic region

In the poro-elastoplastic region stresses are at the yield surface. We have adopted the Mohr-Coulomb yield surface in our semi-analytical model. It is defined as:

$$\sigma_1'^{pl} = -2S_0\sqrt{\gamma} + \gamma\sigma_3'^{pl} \quad \left(\gamma = \frac{1+\sin\phi}{1-\sin\phi}\right)$$

where S_o is the cohesion and ϕ is the friction angle. Using the yield surface and the equation of equilibrium, the stresses in the region is given as:

$$\sigma_{rr} = \sigma_{rr}^A + P^A - P(r) - r^{\gamma-1}\int_{r_A}^{r}\left[\frac{\tilde{Q}}{2\lambda(\rho)} + 2S_0\sqrt{\gamma}\right]\frac{d\rho}{\rho^\gamma}$$

$$\sigma_{\theta\theta} = -2S_0\sqrt{\gamma} + \gamma\sigma_{rr} + (\gamma-1)P(r)$$

The deformation in the region is calculated using a non-associative flow rule. The validation of the proposed solution is shown in the **Fig. 2** for the parameters given in **Table 1.**

A preliminary result for a more complex plasticity model is shown in **Fig. 3**.

Table 1

Depth [m]	2500	E [GPa]	15
H_{res} [m]	100	K_{fluid} [GPa]	2
R_w [m]	0.1	ν [-]	0.20
k [md]	20	σ_h [MPa]	-40
μ [mPa.s]	0.365	α_{Biot} [-]	1.0
ϕ [–]	0.20	μ_{MC} [–]	0.57
P_0 [MPa]	25	S_0 [MPa]	1.0
T [°C]	80	ψ [-]	0.10
s_f [m]	0	Q_{inj} [m³/min]	0.6

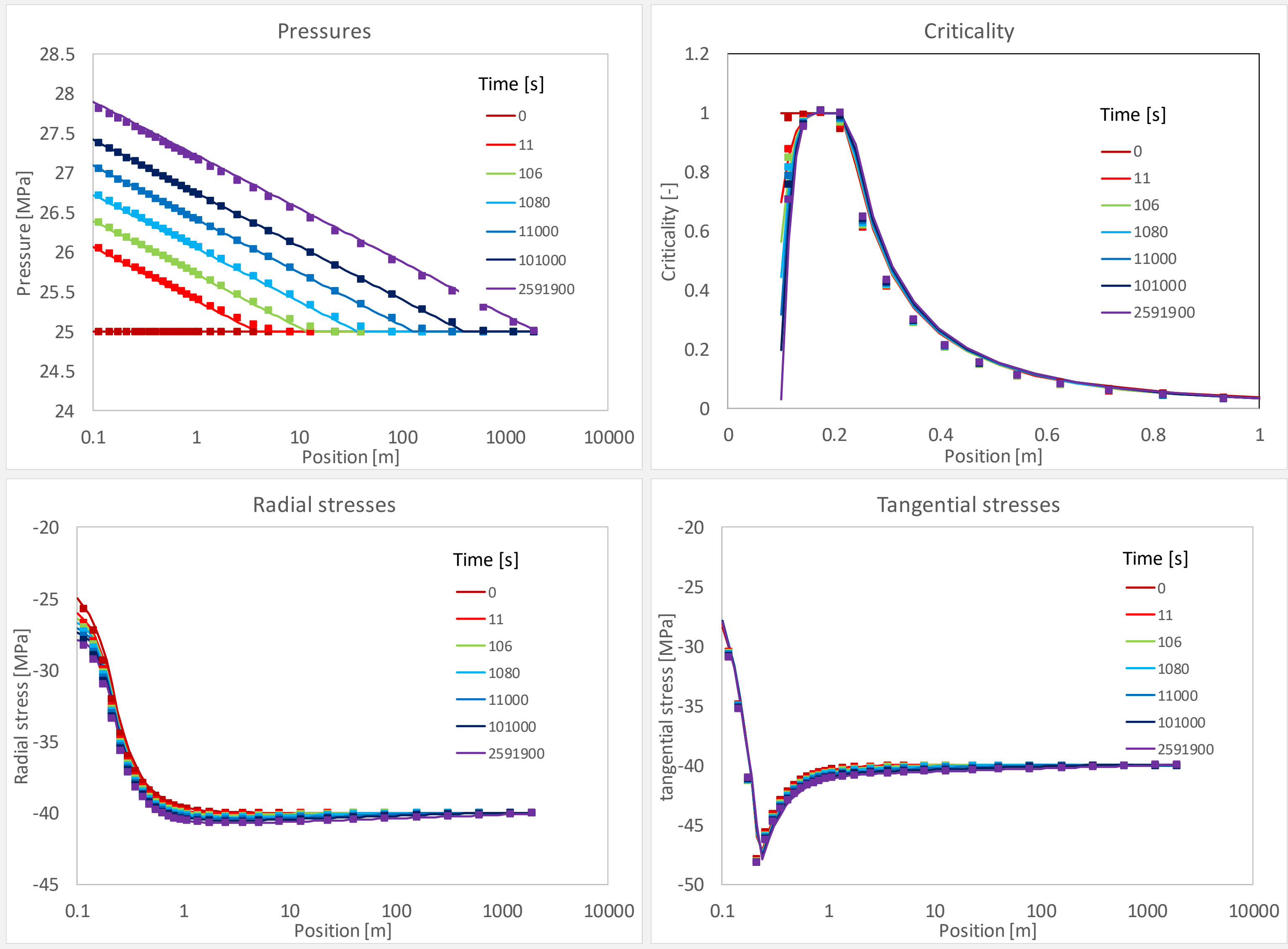

Figure 2
Validation of the pressure development, criticality, radial and tangential stresses for injection case with TOUGH-FLAC. Symbols are calculated with TOUGH-FLAC; line represent the solution from the proposed semi-analytical solution. The results are for parameters given in Table 1.

Note: ***The injection case has been validated using a numerical code, FLAC-TOUGHREACT [Taron and Elsworth, 2019, Fokker and Wassing, 2019]. This code coupled the thermal and hydrologic capabilities of TOUGHREACT with the mechanical framework of FLAC3D, to examine THM processes in deformable fractured media.***

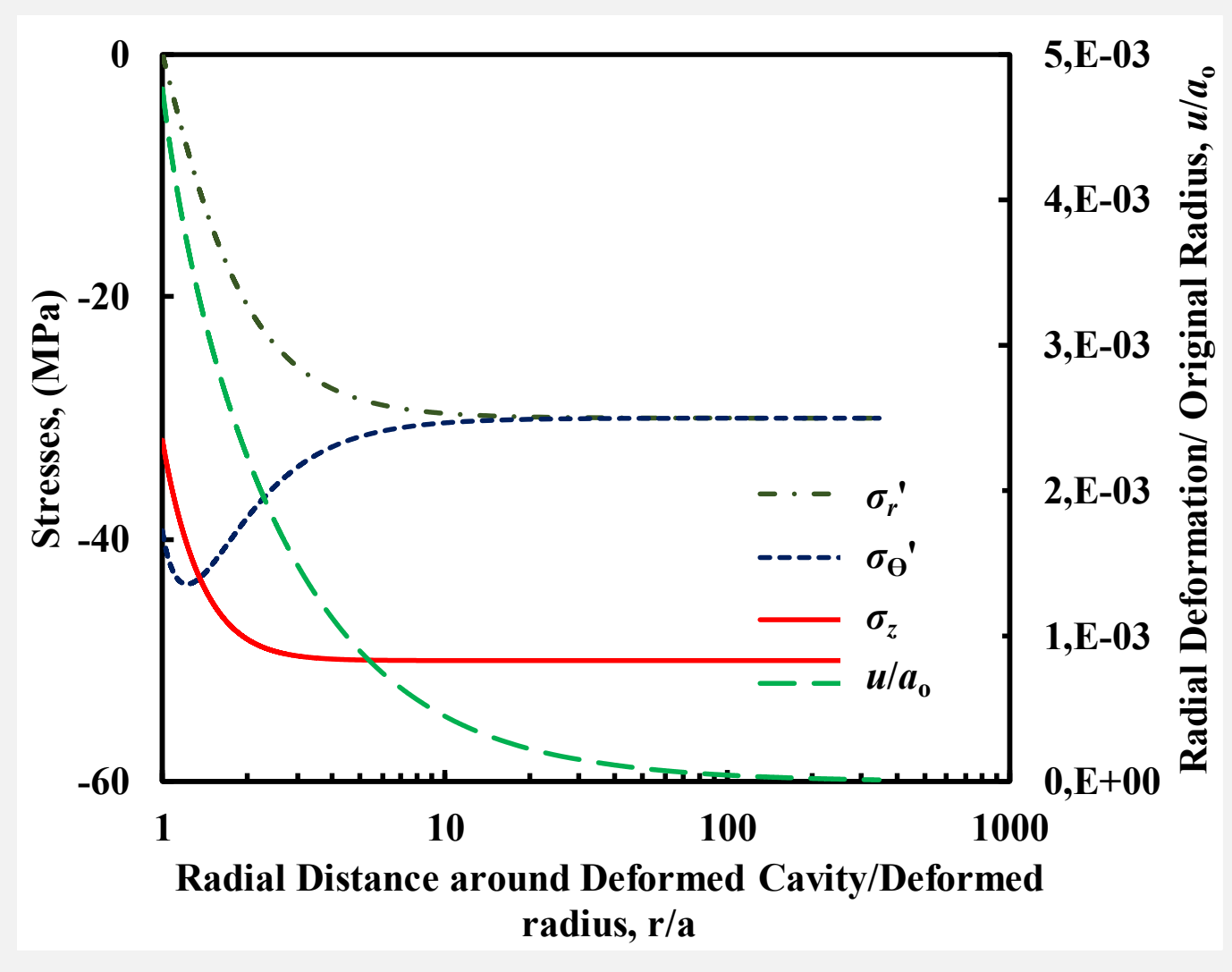

Figure 3
The response of the ground due to the formation of the wellbore considering the complete drain behaviour and hardening behavior proposed by Pijnenburg et al. (2019).

Conclusion

We devised a semi-analytic coupled poro-elastoplastic model for geothermal applications. As it fundamentally employs a sequential time-dependent scheme, it is optimally suitable for time-dependent plasticity, for tracking developing reservoir properties, and for operational scenarios with changing injection or production rates. Plastic behaviour can take place close to the wellbore, but also in isolated regions away from it.

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