Utrecht University

Copernicus Institute
for Sustainable Development

Solution Algorithms for large Energy System Models

based on Warm-starting of Mixed Integer Linear Solvers

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We would like to design an energy system for heat, hydrogen and electricity generation! Can you help us?

Sure, here is what we need to to!

Define the topology and all input data

1) Defining boundary conditions
- Weather Conditions
- Energy/Material Demand Data
- Carbon Limits

2) Technology & Network Selection
- Which Technologies
- Which Networks

3) Defining Economic Data
- Technology Investment/ O&M costs
- Network Investment/ O&M costs
- Energy & Carbon Prices

Here you go with hopefully all you need!

Cool, thanks, looks complete, let me illustrate that for you, real quick!

$$\min_{x,y} \left(c^T x + d^T y\right)$$

subject to

$$Ax + By = b$$

$$x \geq 0 \in \mathbb{R}^N, y \in \mathbb{N}^M$$

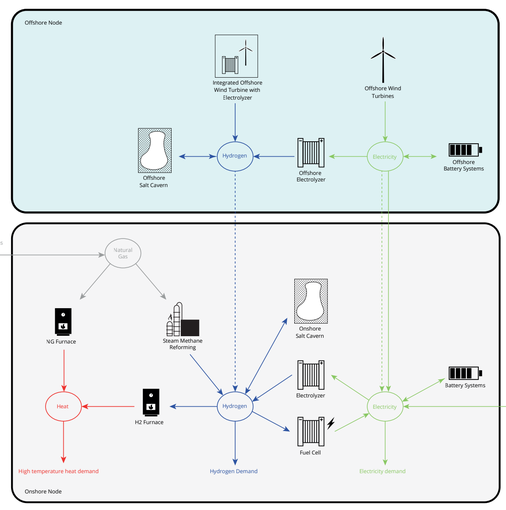
Sure, essentially it consists of a lot of equations and is formulated as a mixed-integer linear program. In contrast to simple linear programs, this means that some decision variables can only take integer values.
If we would use a simple linear program, the optimization might lead us to the conclusion that is optimal to build 3.8 wind turbines offshore - this is clearly impossible.

Some of these problems can easily contain millions of decision variables (as it is the case here) and of course cannot be solved in a pen-and-paper manner. Luckily, we got high-performance computers to do the job for us.

Considering that you want to run a sensitivity analysis, I would guess it takes about 150 hours or so. So talk to you in a week, I'll keep you posted!

Whaaat, so long? Can't you do it quicker?

No, the solution of those problems, are a problem class that is NP-complete.
In other words: it takes a long time to solve it and the computer essentially needs to do a lot of "smart" trial and error. But we all know how smart the computer is...



On the top, you can see the systems topology, including a hydrogen network and an electricity network between an onshore and an offshore node. We give the model a selection of technologies that can be build (or not) depending if it is optimal or not. On the bottom, you can see the essential building blocks of our model that is transferred into a mathematical formulation to determine the optimal design and operation.

In your case, we'll find the least expensive design and operation of the system over a year.

Decision Variables	Input Data

- Design:
  - Technology Selection and Size
  - Network Selection and Size
- Operation:
  - Scheduling (on/off)
  - Operation (part-load, storage,...)
  - Import/Export
	- Weather Conditions
	- Energy Demands
	- Energy Prices
	- Carbon Rates
	- Technology Costs
	- Technology Performances

Constraints	**Objective Function**
- Technology Behavior - Energy Balance	- Total annual costs OR - Total annual emissions

Cool, can you give me a short explanation of what that mathematical model looks like?

180h of work on the University Servers later...

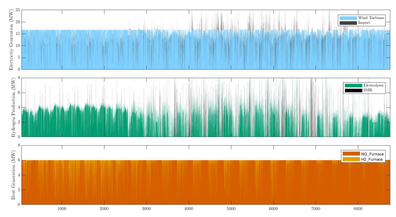
Here are your results. Hopefully next time, we are quicker. We are currently working on a new solution algorithm to combine human and machine intelligence.

Thanks a lot! Looks good. The algorithm sounds interesting though, can you tell me more?

Optimal Design

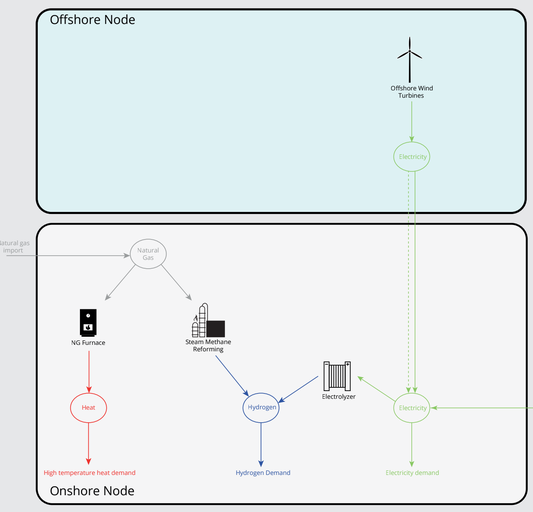
Technology sizes	
Wind Turbine	5
Battery Onshore	25 MWh
Electrolyser Onshore	10 MW
Cavern Onshore	152 MWh
SMR	8 MW
Natural Gas Furnace	6 MW
Hydrogen Furnace	4 MW
Network sizes	
Electricity	50 MW
Hydrogen	0 MW

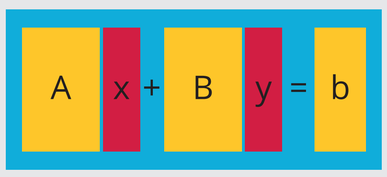
Optimal Operation



Contribution of this work

Stage 1





Solution of the mixed-integer problem:
1) Root relaxation (LP)
  - All integer constraints relaxed to continuous variables
  - Establishes a lower bound on the solution

2) Branch & Bound
  - Finds a feasible solution within a distance to the best bound
  - Iterative, "smart" algorithm
  - Can take very long

The computational time of both steps depends largely on the number of variables.

These mixed-integer linear problems are solved in two steps being described on the left. The larger the systems become and the more detailed we model the technologies, the longer it takes to solve them.

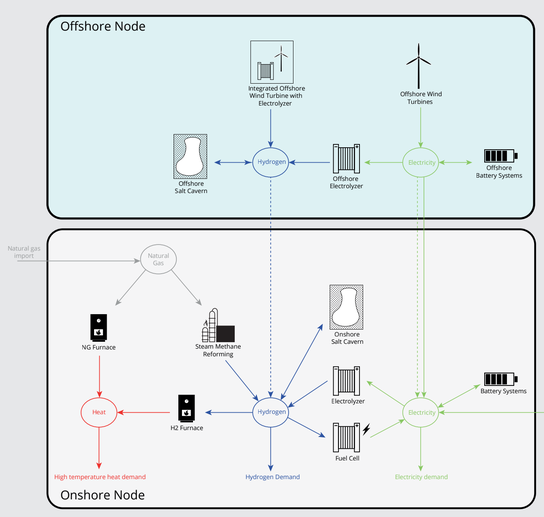
The first step is an important one: since we allow a violation of the integer constraints, the solution from the root relaxation provides a first indication of the minimal possible objective value. If we enforce the integer constraints, we can never be better than that initial objective.

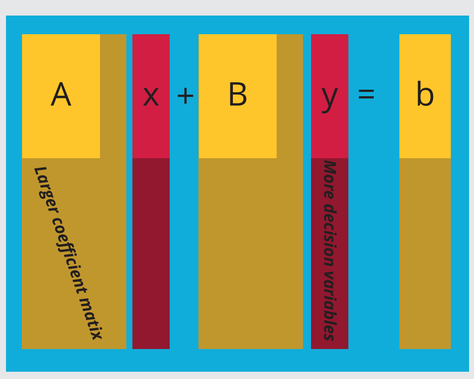
Lets say, we have succesfully solved for the optimal design and operation of the system on the left, but we want to actually solve the system below. What can you deduct for the design and operation of the larger system, given the optimal solution for the smaller system?

Well, isn't it, that the solution we obtained, is also a solution in the larger system? Provided of course that the technologies remain the same...

The optimal solution of stage 1 becomes a feasible solution in stage 2

Stage 2





Exactly! If we provide the solving algorithm with a so called warm-start solution, it can rule out parts of the iterations that are required in the second step.

And even better: If we only solve the root relaxation of the larger system we already have an indication for how better we can actually get by adding technologies to the system - even though we might not have enough resources to solve the larger system. So essentially, we can solve the larger system quicker provided a warm-start derived from a simpler system.

Nice, I think I got the idea. Thanks! What are current challenges you encounter?

Well, we still need to run some experiments to measure the improvements. We do know that the second stage solves quicker provided a warm-start, but we do not know yet how much and what factors influence the time-saving. If you solved a small system that is quite different from the optimum of the large system, the benefits are rather small. So it essentially also depends a lot on the guess of a human to provide the topology of the first stage.

And a story from our daily work: the energy system model we use is written in Matlab, but the functionalities of the solver are only provided in Python - so we are looking at options to create an interface between the two. This means, of course, quite a bit of coding.

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