Understanding quantities in geo-information in terms of amounts, magnitudes, extents, and intents

Eric Top
department of Human Geography and Spatial Planning, faculty of Geo-Sciences, Utrecht University, the Netherlands
Advised by dr. Simon Scheider

Introduction

Quantities are paramount in Geographic Information (GI) science, but their semantics are not well-understood. A GI expert may understand _that_ population counts can and population densities cannot be summed, but they may not understand _why_. To provide the knowledge behind their unspoken intuition, to automate geo-analytical processes and to make quantity data retrievable on the semantic web, a theory of the semantics of quantities in GI is essential. I distinguish two types of quantities, namely *amounts* and *magnitudes*, and I approach them through two modes of reasoning, namely *extensional* and *intensional* reasoning. The aim is to develop principles for modeling GI quantities in computational ontologies.

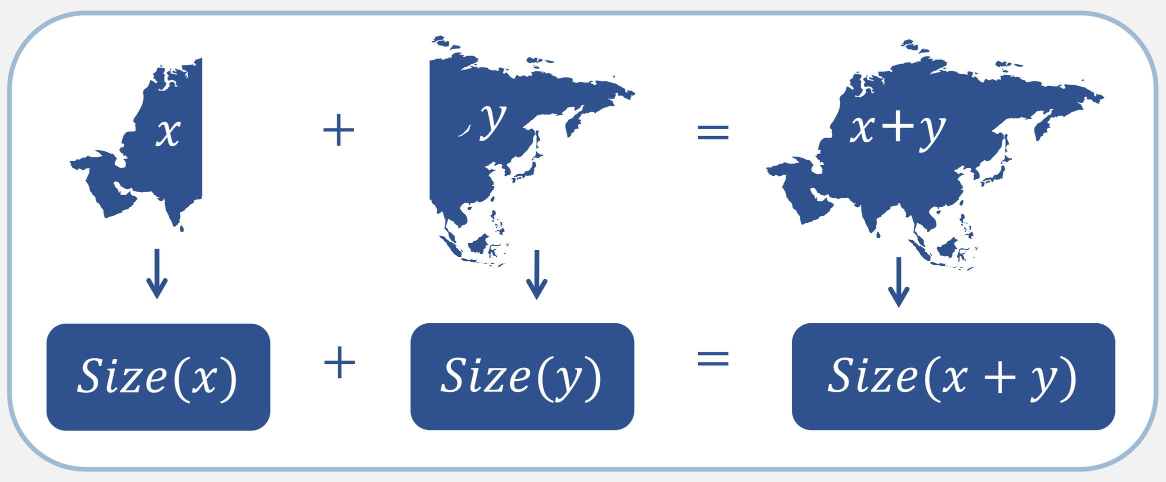
Amount and magnitude measurement

Amounts and magnitudes are quantities in quantity domains. A **quantity domain** is a structure between quantities of the same kind, for example a domain of spatial regions.

Amounts are mereological quantities. They behave like parts in a parthood relation and form a Boolean lattice. **Magnitudes** are vector quantities. They behave according to the axioms of a vector space and form linear orders.

Domains are connected through **measurement functions**. If the measure is an extensive quantity w.r.t. a controlling quantity, then it is directly measured from the control. Otherwise, it is mediated by another amount (See fig. 2).

Extensive quantities are summed when the corresponding entities are aggregated, e.g., the sum of sizes of Europe and Asia is the size of Eurasia.



$$\forall x, y \in X. \neg O(x,y) \rightarrow m(x) + m(y) = m(x+y) \quad \text{(Additivity)}$$
$$\forall x, y \in X. \neg O(x,y) \rightarrow m(x) - m(y) = m(x-y) \quad \text{(Subtractivity)}$$

$O(x,y)$: Do x and y overlap? (e.g., the times 8:20-8:40 and 8:30-8:50 overlap)
$m(x)$: Measurement function on x (e.g., m: $Time \rightarrow Duration$)

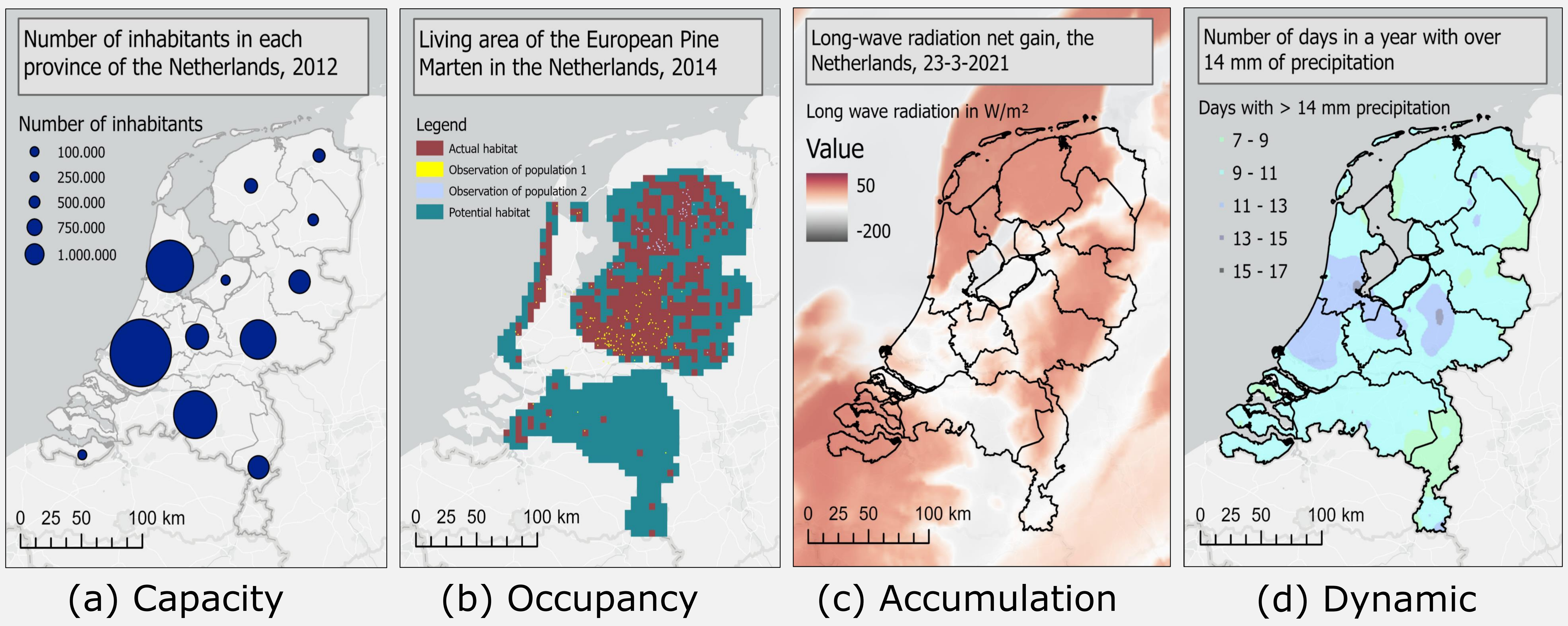


Fig. 1: Examples of measurement functions between amounts

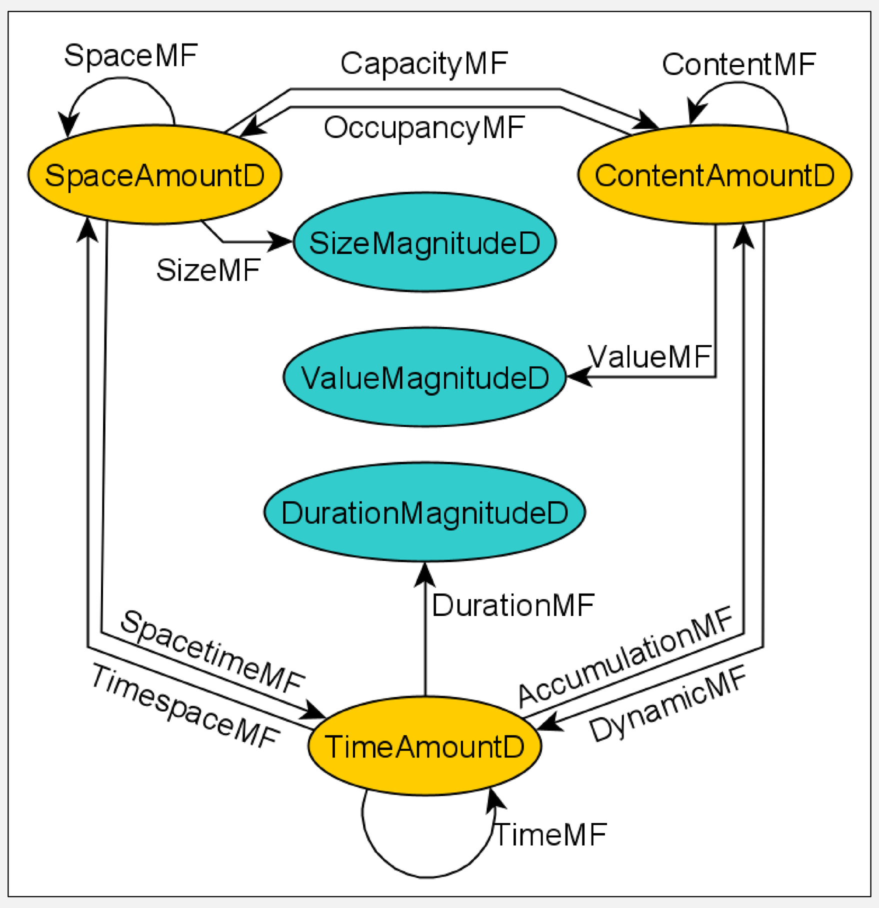


Fig. 2: Measurement functions

Extents and intents

Any term has an extent and an intent. The **extent** encompasses all **examples** and the **intent** encompasses all **characteristics** of a particular term. For example, the term *continent* extends over *Europe* and *Asia* and has *large land mass* in its intent.

Formal concept analysis (FCA) offers a mathematical approach to defining concepts based on these two notions.

Concepts in Formal Concept Analysis (FCA)

Let $A \subseteq X$ and $B \subseteq X$ and $I \subseteq X \times X$. Then:

$\{m \in M \mid \forall g \in A\ ((g,m) \in I)\}$ equals the **intent** of A and
$\{g \in G \mid \forall m \in B\ ((g,m) \in I)\}$ equals the **extent** of B.

A and B form a _concept_ (A,B) of I iff
the intent of A by I equals B and the extent of B by I equals A.

Extents and intents may imply quantities. It is thus possible to *count the atoms in a concept lattice*. Atoms are vertices nearest to but not at the concept lattice's extrema.

Fig. 3 shows parthood relations between spatial raster regions and propertyhood relations with their land uses. Each region has a number of land uses and each land use has a number of spatial regions, depending on the scale level.

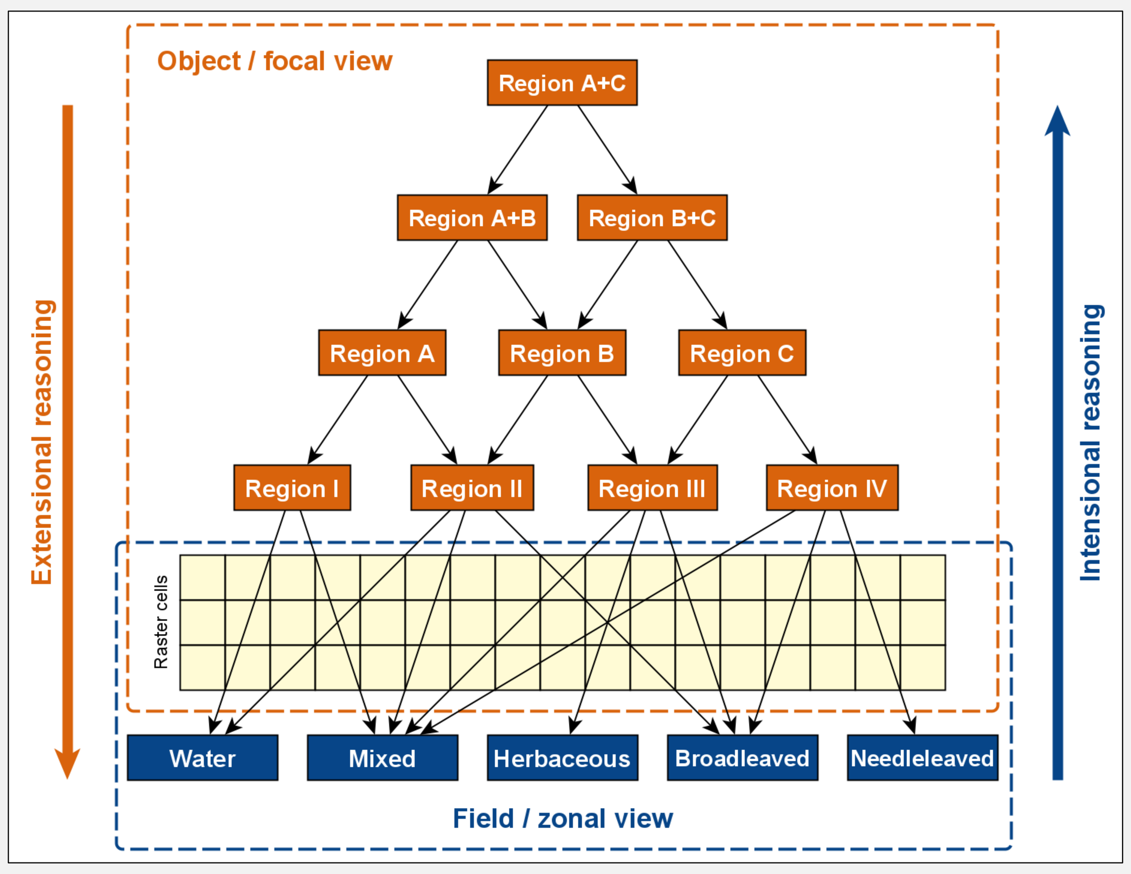


Fig. 3: Extents and intents of raster regions

References

1. Top, E., Scheider, S., Xu, H., Nyamsuren, E., & Steenbergen, N. (2022). The semantics of extensive quantities within geographic information. Applied Ontology, 17(3), 337-364.
2. Top, E., & Scheider, S. (2023). I am What I Have and I Have What I am, What am I?. In Formal Ontology in Information Systems (pp. 210-224). IOS Press.
3. Top, E. J., Romm, D. & McKenzie, G. (2023a), Exploring the Duality of Space and Place through Formal Geo-Concepts, in 'Proceedings of the Fourth International Symposium on Platial Information Science (PLATIAL'23)', Zenodo, Dortmund, Germany.

uu.nl/medewerkers/EJTop