Langevin Regression for a Softglass system

Zach McBrearty, Naud de Boeij, Thomas Muhren, Roberto Benzi, Federico Toschi, Jeannot Trampert

Utrecht University

TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Introduction

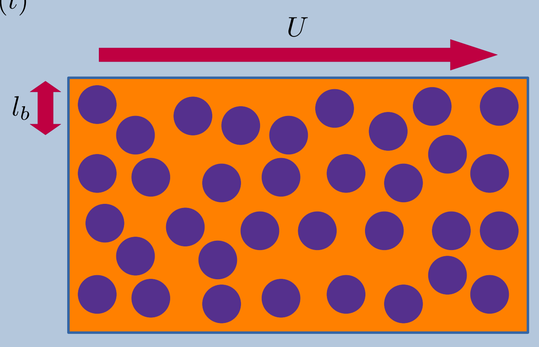
- A softglass is an emulsion, e.g. of oil and water
- It can display a range of behaviours when different kinds and strengths of forcing are applied
  - Viscoelastic wave propagation
  - Flow
  - Stick-slip
  - Shear-banding (localised flow)

We can model the bulk properties of fluidity (inverse viscosity) and stress of the softglass with [1]: a stochastic differential equation for ϕ, the square root of fluidity,

$$\dot{\phi}(t) = -R\phi + m(\sigma)\phi^3 - |\phi|\,\phi^3 + \sqrt{\epsilon_0 + \epsilon_1\phi^2}\eta(t)$$

and a Maxwell relation for the stress σ,

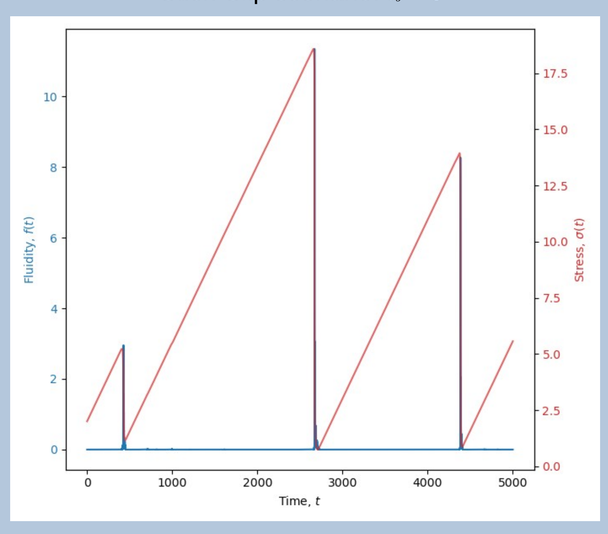
$$\dot{\sigma}(t) = \frac{1}{\tau}\left(U - (1-l_b)\phi^2\sigma - l_b m(\sigma)\sigma\right)$$



- Physically these equations model a Couette flow:
  - Purple droplets: Fluid 1, e.g. oil
  - Orange medium: Fluid 2, e.g. water
  - Shear-strain rate U imposed by moving the top wall
  - A flowing region of size l_b at by the moving wall
  - Periodic walls on the left and right

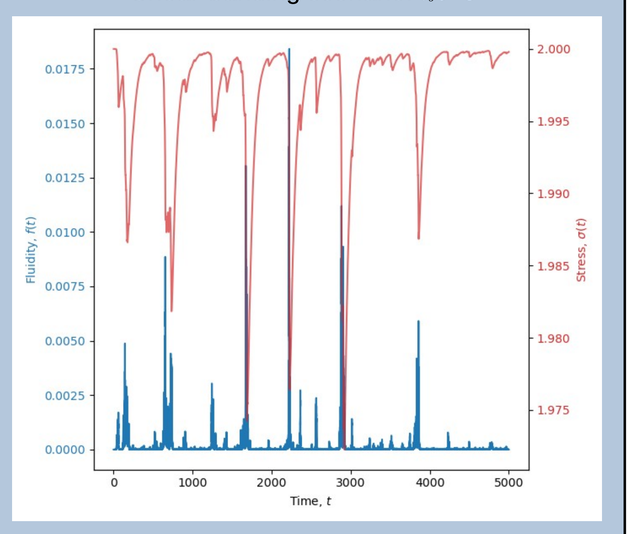
We can evolve these equations for the stick-slip and shear-banding scenarios by tuning the l_b parameter.

Stick-Slip scenario: $l_b = 0$



- Stress can build up infinitely
- An avalanche is a sharp delta-function like event
- Most of the stress in the system is released

Shear-banding scenario: $l_b > 0$

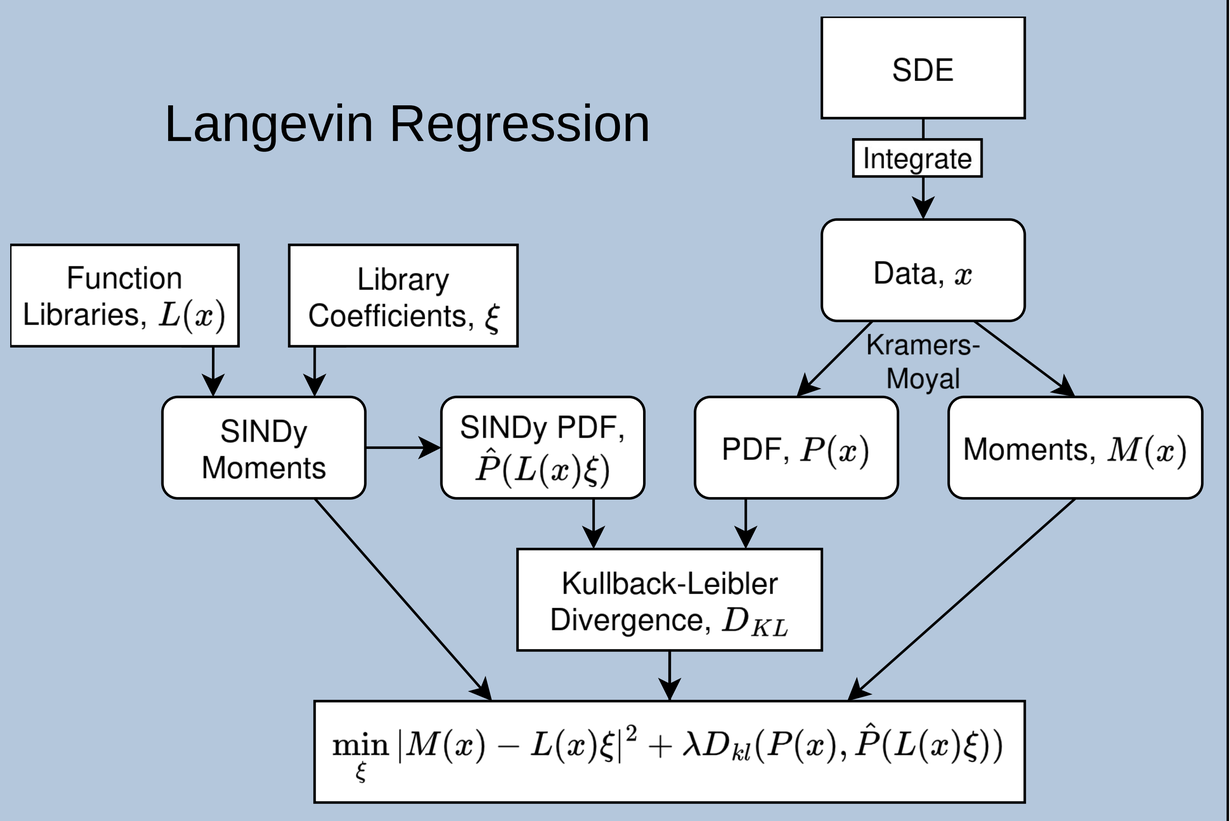


- Maximum stress due to flowing region
- Avalanches last for longer
- Stress drops are small compared to the maximum

Q: Can the SDE be recovered given $\phi(t)$ and $\sigma(t)$?

Yes, but only in certain areas of the parameter space:
- Example 1 shows a more simple version of the SDE with constant stress
- Example 2 shows a regression which performed better due to a lower $m(\sigma)$
- Example 3 shows a lower value of ϵ_0 which causes the regression to fail

Method



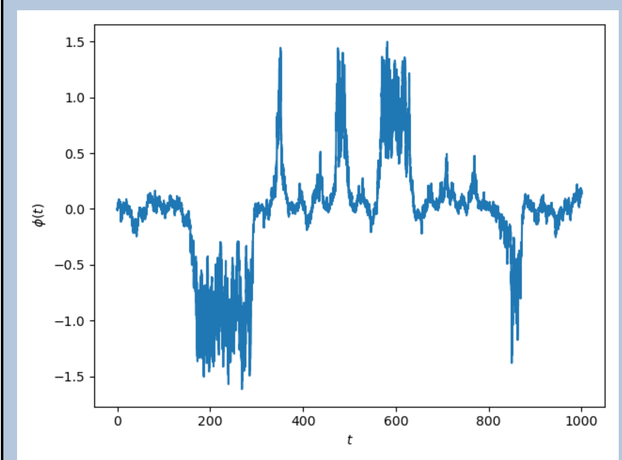
- Stochastic SINDy [2]
  - First moment of the PDF corresponds to the drift (deterministic) part of the SDE
  - Second moment of the PDF corresponds to the diffusion (stochastic) part of the SDE
  - Fit a library $L(x)$ of (non-linear) basis functions with coefficients ξ to those moments
- Langevin Regression [3]
  - Solve the Steady state Fokker-Planck equation for the PDF corresponding to the SINDy moments
  - Regularise the ill-posed problem with the Kullback-Leibler divergence of the data PDF and SINDy PDF
  - Can also find white noise approximations to coloured noise systems and systems with unresolved dynamics

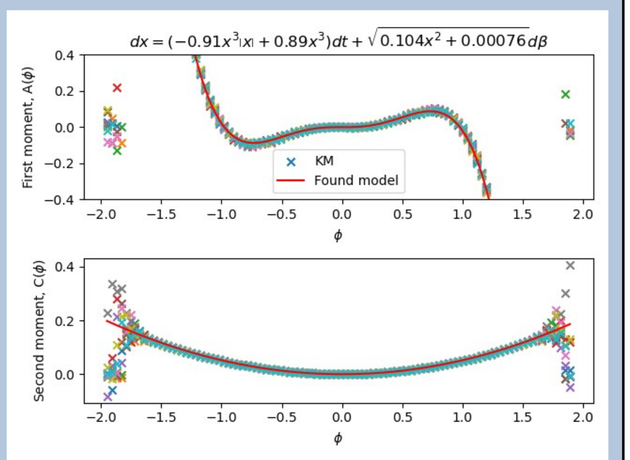
References

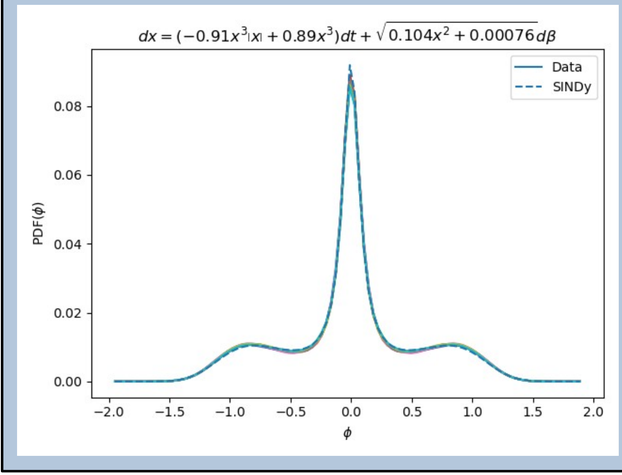
[1] Benzi et al. 2016 Cooperativity flows and shear-bandings: a statistical field theory approach. *Soft Matter*, **12**, 514
[2] Boninsegna et al. 2018 Sparse Learning of stochastic dynamical equations. *J. Chem. Phys.* **148**, 241723
[3] Callaham et al. 2021 Nonlinear stochastic modelling with Langevin Regression. *Proc. R. Soc. A* **477**: 202110092

Example 1: 0D fluidity model with constant stress

- Physically corresponds to controlling the stress instead of the shear-strain rate
- Results in a pitchforking behaviour where the system switches between stable states at 0 and ± 1



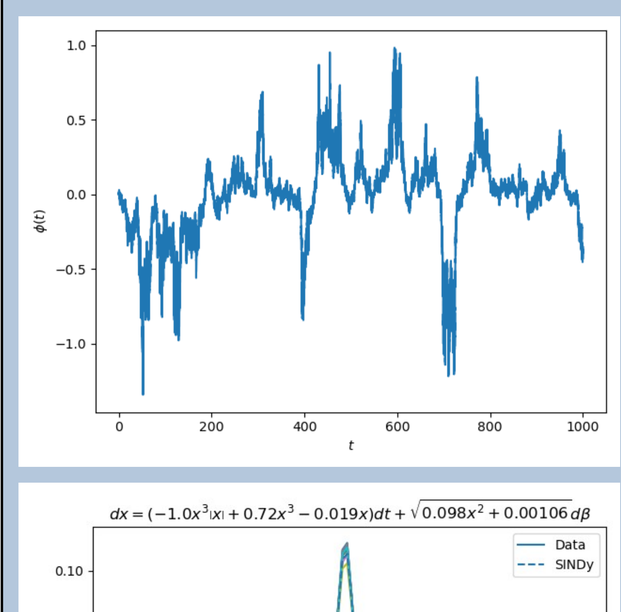


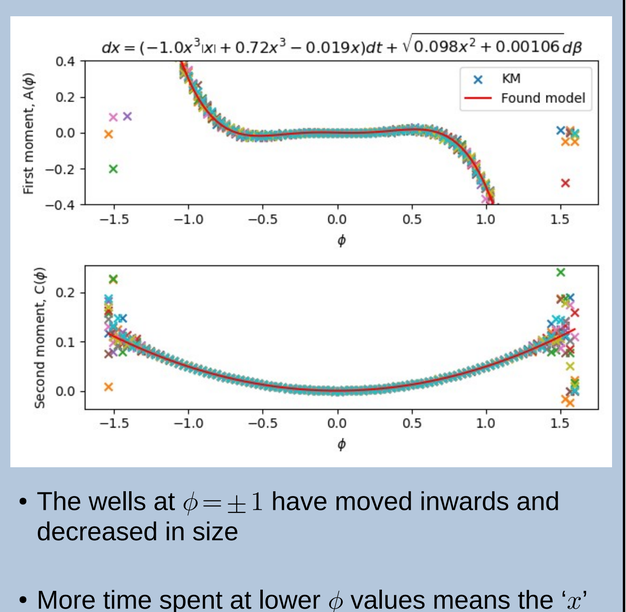


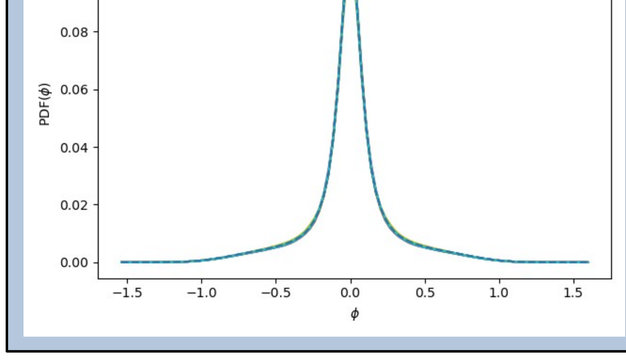
- Found equation is very close to the true model
- The 'x' term is missing from the drift
- The found diffusion for the model is almost exactly correct, in this case $\epsilon_0 = 10^{-3}$ and $\epsilon_1 = 0.1$
- This regression first fitted the PDF, then the correct scaling is found to fit the moments

Example 2: Example 1 with lower value of $m(\sigma)$

- The stress is lowered from $\sigma = 2.6$ to 2.0 ($m(\sigma) = 1.0$ to 0.7) which is closer to the stress seen in the shear-banding scenario



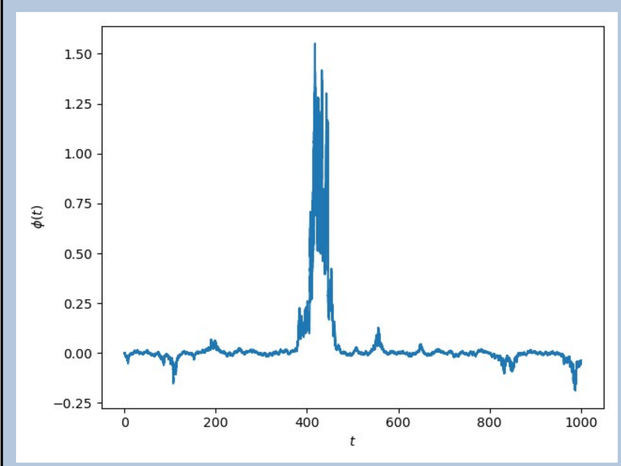


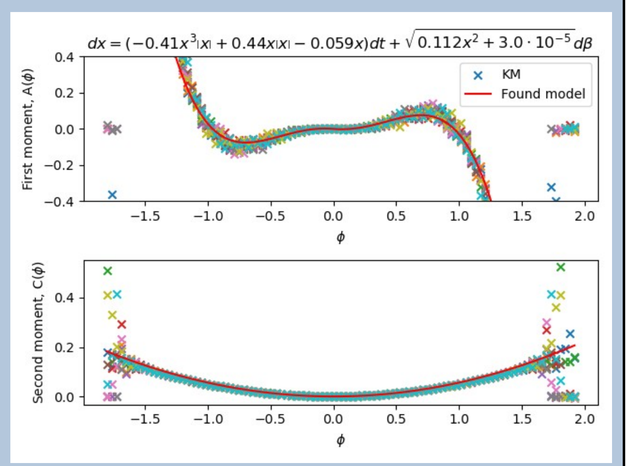


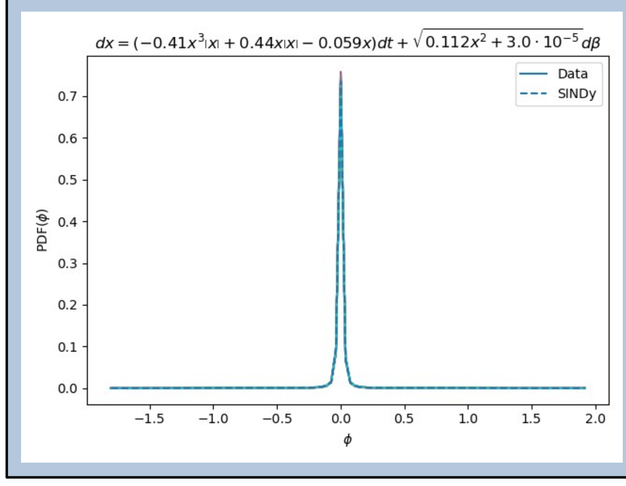
- The wells at $\phi = \pm 1$ have moved inwards and decreased in size
- More time spent at lower ϕ values means the 'x' term is now found
- All the coefficients for the model are very close to their true values

Example 3: Example 1 with lower value of ϵ_0

- The value of ϵ_0 is reduced from 10^{-3} to 10^{-5} resulting in a much thinner pdf around 0, but with the same excited states at $\phi = \pm 1$







- Drift has incorrect terms, missing terms, underestimated and overestimated terms
- Diffusion has both correct terms, with ϵ_0 being overestimated
- Strategies to overcome this problem need to be explored

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